# THREE-DIMENSIONAL FACE STABILITY ANALYSIS OF SHALLOW CIRCULAR TUNNELS

# **Abdul-Hamid Soubra**<sup>1</sup>

### **ABSTRACT**

The upper-bound method of the limit analysis theory is used to calculate the active and passive limit pressure in front of a pressurized shield. Two translational kinematically admissible failure mechanisms are considered for the calculation schemes. The numerical results obtained are presented and compared to those given by other authors.

## INTRODUCTION

Over the past twenty years, tunneling in cohesionless soils has been made possible due to recent technological advances including the pressurized shield. Face stability analysis of shallow circular tunnels driven by the pressurized shield requires determination of the pressure to be applied by the shield to ensure the tunnel's face stability and limit environmental impact of tunneling. This pressure must avoid both the collapse (active failure) and the blow-out (passive failure) of the soil mass near the tunnel face. Active failure of the tunnel is triggered by application of surcharge and self weight, with the tunnel face pressure providing resistance against collapse. Under passive conditions, these roles are reversed and the face pressure causes blow-out with resistance being provided by the surcharge and self-weight.

In this paper, the collapse and the blow-out failures are investigated by the upper-bound theorem of the limit analysis theory using respectively collapse and blow-out mechanisms. These mechanisms allow the slip surface to develop more freely in comparison with the available mechanisms given by Leca and Dormieux (1990). It is to be noted that the upper-bound theorem gives unsafe estimates of the collapse and the blow-out loads. The aim of this work is to improve the available solutions by increasing (respectively reducing) the best available upper-bound solutions given in the collapse (respectively blow-out) case.

# KINEMATICAL APPROACH TO THE FACE STABILITY ANALYSIS

The problem can be idealized, as shown in figures 1 and 2, by considering a circular rigid tunnel of diameter D driven under a depth of cover C. A surcharge  $\sigma_s$  is applied at the ground surface and a constant retaining pressure  $\sigma_t$  is applied to the tunnel's face.

To obtain upper-bound solutions for the collapse and blow-out problems, kinematically admissible failure mechanisms must be considered. According to the normality condition for an associated flow rule Coulomb material, for a kinematically admissible failure mechanism, the velocity along a plastically deformed surface must make an angle  $\phi$  with this velocity discontinuity [cf. Chen (1975)].

## **Failure Mechanisms**

Davis et al. (1980) used the upper bound method in limit analysis and developed solutions for the collapse problem in the analysis of tunnel face stability. These authors considered the case of a Tresca material ( $\phi$ =0°) and their mechanism was composed of two truncated cylinders. Later, Leca & Dormieux (1990) adapted this mechanism to the case of a Coulomb (c,  $\phi$ ) material by replacing the cylinders with cones to respect the normality condition imposed by the upper-bound method of the limit analysis theory. These authors presented solutions for collapse and blow-out problems in

<sup>&</sup>lt;sup>1</sup> Professor, IUP Génie Civil et Infrastructures, L.G.C.N.S.N., Bd. de l'université, BP 152, 44603 Saint-Nazaire cedex, France. Formerly, E.N.S.A.I.S., 24, Bd. de la Victoire, 67084 Strasbourg cedex, France.

the analysis of tunnel face stability. The results they presented showed significant differences with the lower bound solutions presented by Leca and Panet (1988). In the current paper, two failure mechanisms M1 and M2 are considered for the tunnel face stability. For both mechanisms, when the total rate of energy dissipation and the total rate of external work are equated, the pressure  $\sigma_t$  can be expressed as follows:

$$\sigma_t = \gamma D N_{\gamma} + c N_c + \sigma_s N_s \tag{1}$$

Where  $N_{\gamma}$ ,  $N_{c}$  and  $N_{S}$  are non dimensional parameters. They represent respectively the effect of soil weight, cohesion and surcharge loading. Notice that:

$$N_c \tan \phi + 1 - N_s = 0 \tag{2}$$

Hence, in the following, only the  $N_{\gamma}$  and  $N_{s}$  coefficients will be presented; the  $N_{c}$  coefficient can be obtained using (2).

# Collapse Mechanism M1

*M1* is an improvement of the collapse two-block mechanism presented by Leca and Dormieux (1990). It is composed of a shearing zone *OAB* sandwiched between two rigid truncated cones *OACD* and *OBE* (Figure 1).

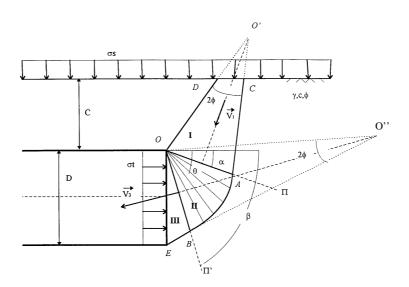


Figure 1 : Failure Mechanism *M1* (Collapse)

The geometrical construction of this mechanism is similar to that of Leca and Dormieux (1990). The upper rigid cone O'OA with a circular cross-section is truncated by the plane  $\pi$ . This cone is called 'Block I'. It will or will not intersect the ground surface depending on the C/D value. On the other hand, the lower rigid cone O''OE with a circular cross-section is truncated by the plane  $\pi'$  and by the vertical tunnel face. This cone is called 'Block III'. It has an axis inclined at an angle of  $\pi/2-\beta$  with the horizontal direction and thus, the intersection of this cone with the vertical tunnel face is an ellipse. This is in conformity with the experimental observations in centrifuge tests as mentioned by Leca and Dormieux (1990). Finally, the shear zone OAB is bounded by a three-dimensional discontinuity surface that is a log-spiral curve AB in the vertical median plane of the tunnel. The equation of the curve, in polar coordinates is  $D(\theta) = D_1 \cdot \exp((\theta - \alpha) \cdot \tan \phi)$  where  $D(\theta)$  is the radius of the log-spiral at an angle of  $\theta$ ,  $D_1$  and  $D_3$  are the radii of the log-spiral at A and B respectively and the center of the log-spiral is D. Note that the cones D'DA and D'DE considered in this mechanism have an opening angle equal to DA0. We will now describe the velocity field for this

mechanism. M1 is a translational mechanism. The different blocks of this mechanism move as rigid bodies. The upper and lower rigid truncated cones OACD and OBE move with the velocities  $V_1$  and  $V_3$  respectively. These velocities are collinear with the cones' axes. This satisfies the condition imposed by the normality condition for an associated flow rule Coulomb material since the velocity jump vector is inclined at an angle  $\phi$  to the discontinuity surface. The shearing zone OAB is composed of infinitely small-truncated rigid cones with circular cross-sections and an opening angle equal to  $2\phi$ . These rigid cones translate with velocities of different directions, which are collinear with the cones' axes and make an angle  $\phi$  with the discontinuity surface. The velocity of each small cone is determined by the condition that the relative velocity between the cones in contact has the direction that makes an angle  $\phi$  with the contact surface. It can be easily shown that the velocity V of each cone is  $V(\theta) = V_1 \cdot exp((\theta - \alpha) \cdot tan \phi)$ . Notice that the log-spiral curve AB is assumed to be tangent to the upper and lower cones at points A and B respectively; thus, there are no velocity discontinuities along OA and OB. As shown in figure (1), this mechanism is completely defined by two angular parameters  $\alpha$  and  $\beta$ . We will now present the work equation for this kinematically admissible mechanism. The external forces contributing to the rate of external work consist of (i) the self-weight of the upper and lower truncated rigid cones (Block I and III) and of the shear zone OAB; (ii) the surcharge loading  $\sigma_s$  (in case of outcrop of the upper rigid block) and (iii) the pressure  $\sigma_t$  at the face of the tunnel. The rate of energy dissipation occurs along the lateral surfaces of the upper and the lower rigid cones. It also occurs along the lateral surface and the radial planes of the radial shear zone. By equating the total rate of external work to the total rate of internal energy dissipation, one obtains an equation, which has the same form as equation (1).

### Blow-out Mechanism M2

Even though safety against collapse is a major concern during tunneling, the blow-out mechanism may be of interest for very shallow tunnels bored in weak soils, when the pressure  $\sigma_t$  can become so great that soil is heaved in front of the shield. Such phenomenon has been observed during tunneling projects (Clough et al. 1983).

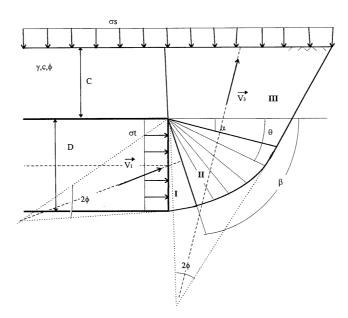


Figure 2 : Failure Mechanism *M2* (Blow-out)

M2 is a blow-out mechanism (cf. figure 2). It represents the passive case of the former mechanism. It is composed of three zones. Zones 1 and 3 are two rigid blocks (Blocks I and III respectively) consisting of two truncated rigid cones moving upward with velocities  $V_1$  and  $V_3$  respectively. The radial shear zone (Zone 2) is limited by a log spiral in the vertical symmetrical plane of the tunnel. With reference to M1, the M2 mechanism presents an upward movement of the three zones, thus, the cones with an opening angle  $2\phi$  are reversed and the log spiral's radius increases with the  $\theta$ -decrease. Consequently, the velocity along the log-spiral of the shear zone is given by:  $V(\theta) = V_1 \exp((\beta - \theta) \tan \phi)$ . Contrary to M1, the present mechanism always outcrops. It is

completely defined by two parameters  $\alpha$  and  $\beta$  as shown in figure (2). As for the M1 mechanism, by equating the total rate of external work to the total rate of internal energy dissipation, one obtains an equation, which has the same form as equation (1).

## NUMERICAL RESULTS AND DISCUSSIONS

By equating the total rate of external work to the total rate of internal energy dissipation for both the M1 and M2 mechanisms, one obtains the  $N_{\gamma}$  and  $N_{s}$  factors as function of the two angles  $\alpha$  and  $\beta$ . To obtain the critical  $N_{\gamma}$  and  $N_{s}$  corresponding to the blow-out case (respectively the collapse case), one has to minimize (respectively maximize) these factors with respect to the  $\alpha$  and  $\beta$ -angles.

As mentioned before, Leca and Dormieux (1990) have considered a collapse failure mechanism composed of two rigid cones and forced the upper cone to remain vertical. We present in figures (3) and (4) the  $N_{\gamma}$  and  $N_{s}$  values given by the present analysis (Mechanism  $M_{I}$ ) and the ones given by Leca and Dormieux.

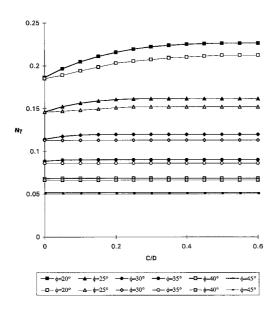


Figure 3 : Comparison of Present  $N_{\gamma}$  (thick lines) with that of Leca and Dormieux (1990) (thin lines) [Collapse]

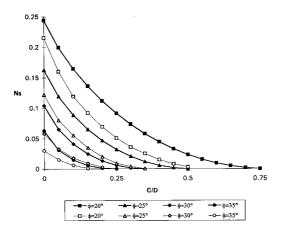


Figure 4 : Comparison of Present  $N_s$  (thick lines) with that of Leca and Dormieux (1990) (thin lines) [Collapse]

It is clear that  $N_{\gamma}$  increases with C/D; then, it becomes constant for larger values of C/D corresponding to the condition of no-outcrop of the upper block. However,  $N_s$  decreases with the C/D-increase and vanishes beyond a certain value of C/D corresponding to the outcrop condition. In this case, the surcharge loading will have no influence on the critical  $N_s$  value. These conclusions conform to those of Leca and Dormieux (1990). It should be mentioned that the present failure mechanism gives greater upper-bound solutions than the best available upper-bound solutions proposed by Leca and Dormieux (1990). The improvement of the solution is about 8% for the  $N_{\gamma}$  factor when  $\phi$ = $20^{\circ}$  and C/D>0.55.

For the blow-out case, Leca and Dormieux (1990) have considered a mechanism composed of a single rigid cone moving upward. The upper-bound solutions presented by them are compared with the ones corresponding to the present *M2* mechanism in figures (5) and (6).

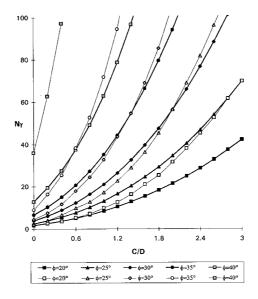


Figure 5 : Comparison of Present  $N_{\gamma}$  (thick lines) with that of Leca and Dormieux (1990) (thin lines) [Blow-out]

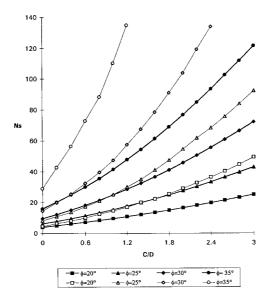


Figure 6 : Comparison of Present  $N_s$  (thick lines) with that of Leca and Dormieux (1990) (thin lines) [Blow-out]

The M2 mechanism is better than the one presented by Leca and Dormieux since the present upper-bound solutions are smaller. For the  $N_{\gamma}$  values, the reduction is very significant and it is of 41% when  $\phi=30^{\circ}$  and C/D=1.4. For the  $N_{s}$  values, significant reductions are also obtained with regard to the results presented by Leca and Dormieux. For example, when  $\phi=30^{\circ}$  and C/D=1.4, the reduction attains 51%.

On the other hand, centrifuge tests have been carried out in Nantes (France) to study the face stability of tunnels in case of collapse (Chambon & Corté 1989). Two soil conditions were examined. A loose sand ( $\gamma=15.3kN/m^3$ ) and a dense sand ( $\gamma=16.1kN/m^3$ ). Shear tests on these soils has shown that c'=2.3kPa,  $\phi'=35.2^{\circ}$  for the loose sand, and c'=1.1kPa,  $\phi'=38.3^{\circ}$  for the dense sand.

 $\begin{array}{c|cccc} C/D & \gamma [kN/m^3] & \sigma_t [kPa] & \sigma_t [kPa] \\ & & (Present\ analysis) & (Centrifuge\ tests) \\ 2 & 15.3 & 3.5 & 4.4 \\ \end{array}$ 

16.1

Table 1: Comparison between the present results and those of the centrifuge tests

The results obtained by Chambon and Corté (1989) are presented in Table (1) and compared to those given by the present analysis. Note that the  $\sigma_t$  given by the present analysis is calculated from direct maximization of this pressure (i.e. there is no error induced by the application of the superposition effect in equation 1). In order to be rigorous, no comparison is made with Leca and Dormieux (1990) in the present section; the calculation of their  $\sigma_t$  value from the corresponding N factors may lead to approximate results due to superposition effect which may err any conclusion. As we can see, there is good agreement between experimental and theoretical results.

### **CONCLUSIONS**

The analysis of translational failure mechanisms has shown that the present mechanisms give better results than the ones considered by Leca and Dormieux. The comparison of our results with those given by Leca & Dormieux (1990) has shown that the present theoretical model improves  $N_{\gamma}$  factor by 8% in case of collapse when  $\phi$ =20° and C/D>0.55; the improvement attains 41% in the blow-out case when  $\phi$ =30° and C/D=1.4. On the other hand, the comparison with the results given by centrifuge tests has shown good agreement.

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