

Deterministic and probabilistic seismic analyses of a slope-footing system

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ABSTRACT

This paper details deterministic and reliability-based analyses at the ultimate limit state of a slope-footing system. The seismic effect is simulated by a pseudo-static approach. The deterministic analysis aims at determining the safety factor F_s of the soil-footing system while the probabilistic one is devoted to the computation of the Hasofer-Lind reliability. The deterministic models used for both deterministic and probabilistic analyses are based on the upper-bound approach in limit analysis using three failure mechanisms. The deterministic results obtained from the limit analysis are validated by comparison with those obtained from numerical simulations based on the finite element software PLAXIS. Also, a parametric study is performed to analyze the effect of the different geometrical and mechanical parameters on both the safety factor and the reliability index.

INTRODUCTION

The geologic structure of slopes in the Lebanese environment is very heterogeneous. The fast urban expansion with the presence of hilly topography pushed the population to build on slopes, therefore enhancing the risk of human induced slope instability. On the other hand, Lebanon is known for being located on a seismic zone. These factors have initiated this research about the seismic deterministic and probabilistic analyses of foundations situated on slopes.

The bearing capacity of a shallow strip foundation located on a horizontal surface has been widely investigated using the limit analysis approach [Michalowski and Dawson (2002), Soubra (1999)]. On the other hand, the literature is scarce when the foundation is on slope and subjected to seismic or different loading conditions. The primary studies available related to the analysis of foundation on slope go back to Giroud and Tran-Vo-Nhiem (1971). Since then, a series of real scale experiments were conducted [e.g. Marechal et al. (1998)]. Recently, the work of Soubra et al. 2004

consisted of developing a model based on the upper boundary of the limit analysis of bearing capacity. Nevertheless, the work remained limited, since it only considered a very high slope where the failure surface can only pass above the toe. Besides, the seismic effect was not considered. On the other hand, the traditional analysis and sizing of geotechnical work are based on the deterministic approach. In this approach, the risks and uncertainties of the different parameters (soil properties, loading, etc.) are considered in a simplistic manner under the global safety factor. In order to account for the risks and uncertainties associated with the parameters used, the reliability theory is getting more and more momentum in geotechnical engineering. These approaches are getting to be very popular among researchers for the analysis of slope stability [i.e. Bhattacharaya et al. (2003), Griffiths and Fenton (2000),] for the slope stability analysis with seismic loading [Youssef Abdel Massih et al. (2010), Al-Homoud and Tahtamoni (2002)], for the behavior of shallow foundations (Soubra and Youssef Abdel Massih (2010), Nevertheless, almost none of the authors have applied these approaches on the seismic analysis of foundations located on slopes.

This paper aims at highlighting the seismic reliability of the behavior of foundations located on the near slopes, taking into account the uncertainties associated with the different parameters used. The seismic effect is modelled by a pseudo-static approach through the use of the horizontal seismic coefficient K_h given as a percentage of the gravity acceleration. Two deterministic models are proposed. The first one is based on the upper bound method of limit analysis. The second one used to validate the first model, is based on numerical simulations using the finite element software PLAXIS. The computation of the reliability will concern only the limit analysis model.

DETERMINISTIC MODELS

Limit analysis models – Failure mechanisms. The deterministic models used for the reliability analysis are based on the upper bound method of limit analysis using three failure mechanisms.

The first one (M1 Mechanism) is a translational mechanism that may pass by, below or above the toe of the slope (Figures 1a and b). This type of failure may occur when the footing is situated near the slope and thus, it has an influence on the slope instability. Soubra et al. (2004) proposed the translational mechanism (M1 Figure 1a) that may pass above the toe of the slope when the footing is situated near the slope. However, this mechanism didn't take into account the influence of the slope height when elaborated in 2004. Thus, the failure was not allowed to pass by or below the toe. In this paper, this mechanism is developed in order to account for the slope height, and is allowed to pass by or below the slope toe by adding the new failure block $A_1A_2A_3$ as shown in figure 1b.

The second failure mechanism is also a translational one developed by Soubra (1999) for the case of a foundation situated on a horizontal soil. The type of failure for this mechanism is symmetrical for the case of a vertical central load (Figure 2a, M2 mechanism) and non-symmetrical for the case of an inclined or seismic load (Figure 2b, M3 Mechanism). This failure may occur when the footing is far from the slope and the soil punching under the footing is more critical than the slope instability.

The third failure mechanism involves a rotational failure bounded by a log-spiral slip surface passing by or below the toe (Figure 3, M4 mechanism). This mechanism may occur when both conditions are satisfied: (i) the footing is placed far from the slope, and (ii) the slope instability is the predominant mode of failure compared to the footing punching.

In this paper, the minimal safety factor F_s and the minimal reliability index deduced from all these types of failure is considered. The safety factors of each mechanism are obtained using the shear strength reduction method by dividing c and $\tan\phi$ by F_s in the energy equation of the upper bound limit analysis method. For given slope and foundation geometries, soil characteristics and a prescribed applied seismic or non seismic load, the safety factors obtained from all these mechanisms are compared together and the smallest (i.e. most critical one) is adopted.

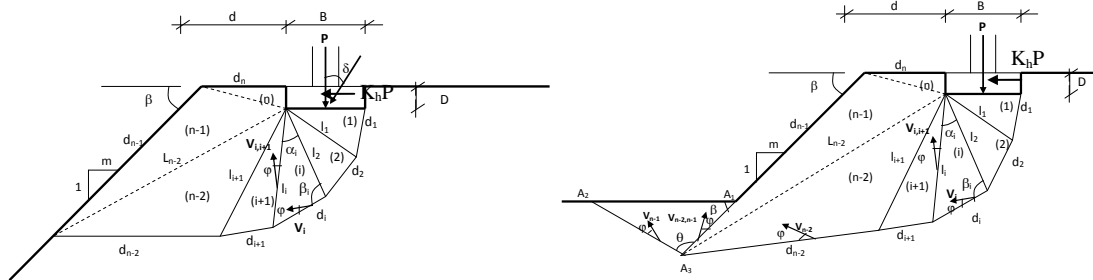


Figure 1. M1 Mechanism passing a) above the toe b) below the toe of the slope

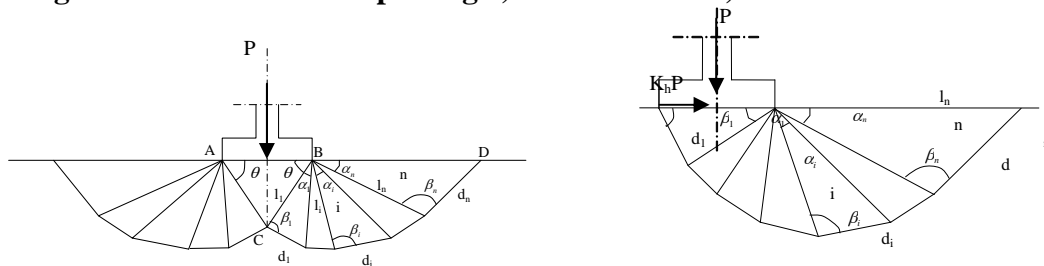


Figure 2. a) Translational symmetrical failure mechanism (M2); b) Translational non-symmetrical failure mechanism (M3)

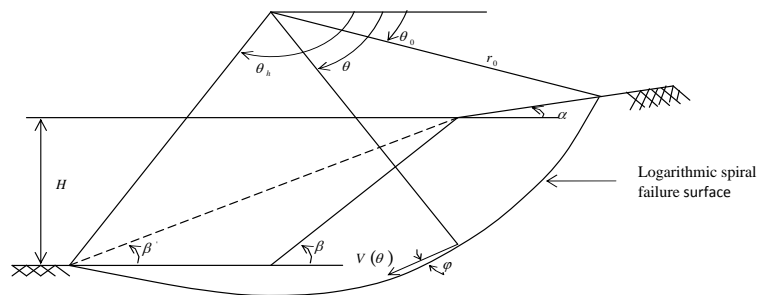


Figure 3. Logarithmic spiral failure mechanism (M4) for the case of a simple slope

Finite element model. In order to validate the limit analysis models (LA), the deterministic results obtained using the failure mechanisms presented above are compared to finite element numerical simulations performed using Plaxis software. The mesh used in the finite element model is optimized. The footing is modeled by an elastic rigid plate element and the soil by a Mohr Coulomb model for which the

dilation angle is chosen equal to the soil friction angle in order to allow a reasonable comparison between the two models (LA and Plaxis). The shear strength reduction technique is used for the calculation of the safety factor.

PROBABILISTIC MODELS

Reliability index, performance function and random variables. The widely used reliability index is the one defined by Hasofer and Lind (1974). Its matrix formulation is given by:

$$\beta_{HL} = \min_{x \in F} \sqrt{(x - \mu)^T C^{-1} (x - \mu)} \quad (1)$$

in which x is the vector representing the n random variables, μ is the vector of their mean values, C is their covariance matrix and F is the failure region. The minimization of equation (1) is subjected to the constraint $G(x) \leq 0$ where the limit state surface $G(x) = 0$ separates the n -dimensional domain of random variables into two regions: A failure region F represented by $G(x) \leq 0$ and a safe region given by $G(x) > 0$ where $G(x)$ denotes the performance function.

The performance function G of the reliability-based approach makes use of the safety factor F_s defined with respect to the soil shear strength parameters c and $\tan(\phi)$ as follows:

$$G = F_s - 1 \quad (2)$$

F_s is equal to the ratio between the maximal shear stress and the mobilized one.

The random variables used in the analysis are: the cohesion c , the angle of internal friction ϕ , the vertical applied load P_s and the seismic coefficient K_h . The cohesion, the friction angle, and the vertical applied load are modeled by the use of a Lognormal distribution where the seismic coefficient is considered to follow an Extreme Value Distribution (Youssef Abdel Massih et al. 2008).

RESULTS

Deterministic results. *Without seismic loading $K_h=0$.* In order to validate the results of the limit analysis failure mechanism models, the static safety factors obtained by these models and those determined by Plaxis simulations are plotted as a function of d/B for different values of H/B in figure 4, where d is the distance from the footing edge to the slope, H is the height of the slope and B is the width of the footing. The soil properties considered in the analysis are as follows: cohesion $c=20\text{kPa}$, friction angle $\phi=30^\circ$, $B=2\text{m}$ and the unit weight $\gamma=18\text{kN/m}^3$. From figure 4, one can notice a good agreement between the two results. It is found that the safety factor tends to a constant value when $d/B > 4$. For $H/B=2, 3$ and 4 , this value is equal to the safety factor obtained by the M4 logspiral failure mechanism of the simple slope. This case corresponds to the situation where the footing is far enough from the slope and when the safety factor of the slope is more critical than that obtained from the M2 mechanism (case of a foundation resting on a horizontal soil). However, for a small value of the slope height when $H/B=1$, the asymptotic value of the safety factor is

obtained using the M2 mechanism. For this case, since the slope height is small and d/B is high, the mechanism M2 is more critical than M4. For $d/B < 4$, the M1 mechanism is found to be the most critical one and is used for the safety factor calculation.

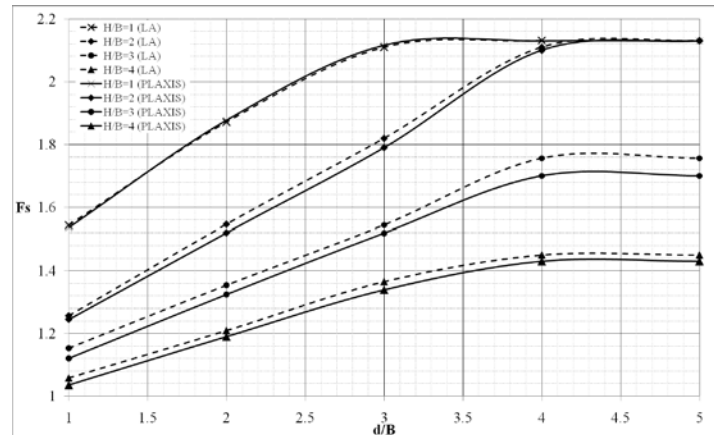


Figure 4. Variation of F_s with d/B and H/B for the limit analysis models and Plaxis simulations for $K_h=0$

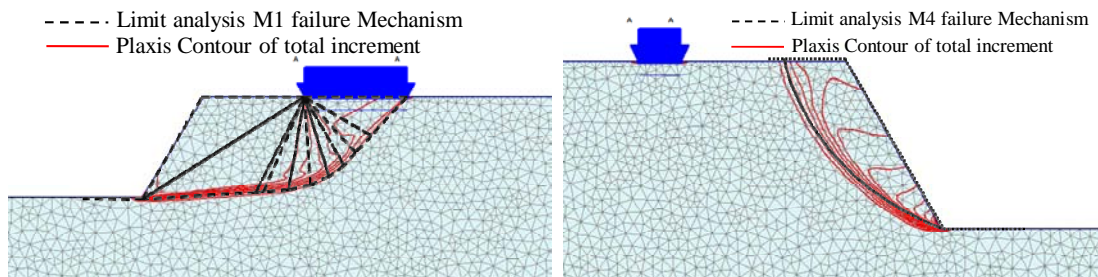


Figure 5. Failure surfaces obtained from the limit analysis and PLAXIS models

A comparison between the failure surface obtained from the limit analysis mechanisms and Plaxis contour of total incremental displacement is presented in figure 5a and b for two cases: (i) when the M1 mechanism is found to be the critical one for $H/B=1$ and $d/B=1$, (ii) when the M4 mechanism is critical for $H/B=4$ and $d/B=4$. Good agreement between the two models is obtained.

With seismic loading. Figure 6 presents a comparison between the pseudostatic safety factors K_h obtained from the Limit analysis models and Plaxis numerical simulations. It is noticed that the LA safety factors are slightly higher than those obtained by Plaxis model. The maximum percent difference is equal to 3%.

Probabilistic results. For the probabilistic results, concerning the statistical parameters of the random variables, a reference case was chosen as follows: for the mean values $\mu_c = 20$ kPa, $\mu_\phi = 30^\circ$, $\mu_{K_h} = 0.15$, $\mu_{p_s} = 300$ kN, and for the coefficients of variation $COV_c = 40\%$, $COV_\phi = 10\%$, $COV_{K_h} = 40\%$, $COV_{p_s} = 40\%$ (Youssef Abdel Massih et al. 20008). The critical reliability (i.e. the

smallest one) obtained among the 3 cases of failures described above is considered as the reliability index of the slope-footing system.

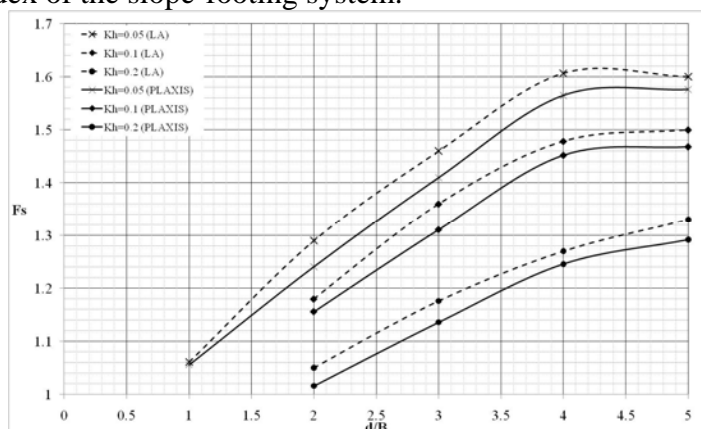


Figure 6. Variation of F_s with d/B and H/B for the limit analysis models and Plaxis simulations with seismic loading

Effect of d/B and H/B on the reliability index. Figure 7 presents the reliability index as a function of d/B and H/B when no seismic loading exists. It is found that when d/B increases for a small H/B ($H/B=1$), the critical reliability index is obtained using the M2 mechanism for the case of a foundation resting on a horizontal soil. However, for higher values of H/B , when d/B increases (higher than 4 for the case of $H/B=3$), β_{HL} tends to the value obtained from the M4 mechanism of a simple slope. This limit value of $d/B=4$ is the same value obtained from the deterministic results.

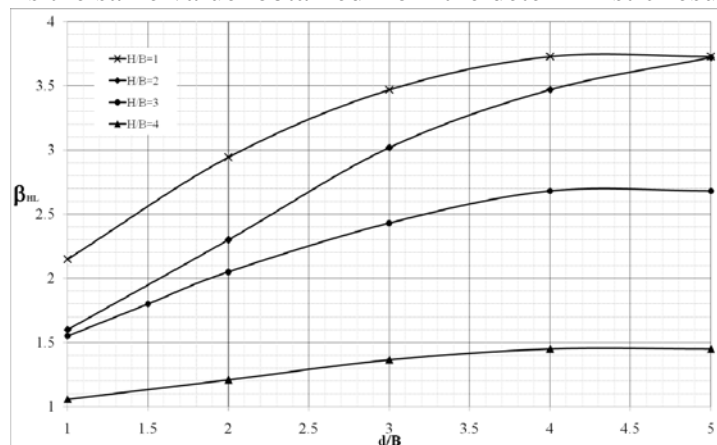


Figure 7. Variation of β_{HL} with d/B and H/B without seismic loading

Effect of K_h and P_s on the reliability index. Figure 8 presents the reliability index as a function of K_h for different values of P_s when $d/B=1$ and $H/B=3$. When K_h increases and P_s decreases, the critical failure tends to be produced in the slope only (i.e. M4 mechanism). This is presented by the 2 upper curves in the figure where we can notice that the reliability index calculated using M4 mechanism is smaller i.e. more critical than the one obtained by the M1 mechanism. The results of the two mechanisms are close to each others when K_h is small. For higher P_s values, the M1 mechanism was found the most critical one.

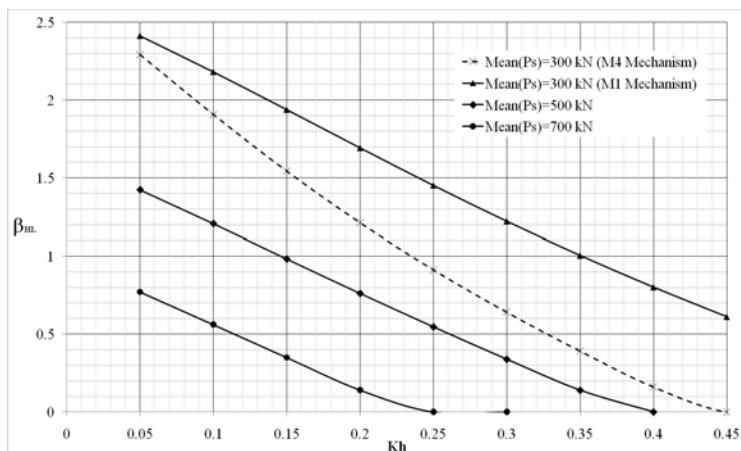


FIG. 8. Variation of β_{HL} with K_h and P_s

Effect of the random variable uncertainties on the reliability index. Figure 9 shows the effect of the coefficient of variation of each random variable on the reliability of the slope-footing system for $d/B=1$ and $H/B=3$. Each curve corresponds to a variation of COV from 5% till 60% for a given random variable. The results of the M1 and M4 mechanism are presented. The results of the M2 mechanism are found too high compared the M1 and M4 (i.e. less critical) and are not presented in this figure. From figure 9, one can notice that for the given soil and load variability, the reliability index obtained from the M1 mechanism is more critical than the one obtained from the M4 mechanism. However, the results of the M1 mechanism are kept on the graph in order to understand the effect of the variability of each parameter on the reliability index obtained by mechanism M1.

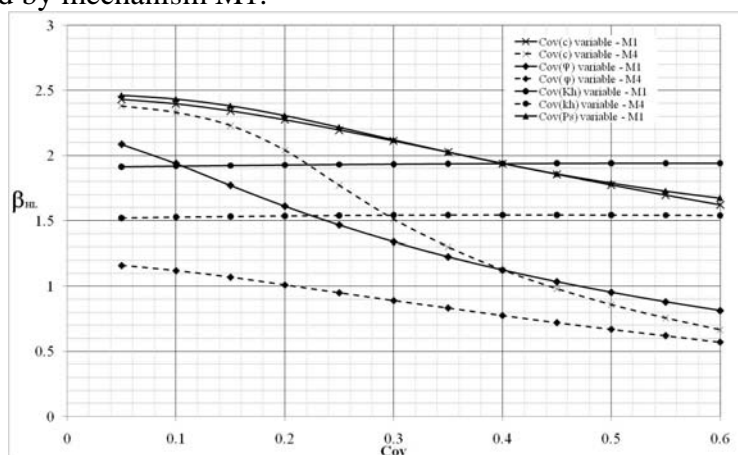


Figure 9. Variation of β_{HL} with the coefficient of variation

For mechanism M1, the variation of c , ϕ , and P_s affect highly the reliability index and have approximately the same degree of influence on the slope-footing system reliability. However, the variation of K_h slightly affect the reliability of the system obtained either using the M1 or the M4 mechanism. For mechanism M4, the effect of the variation of c is very important in the determination of the reliability index (Youssef Abdel Massih et al. 2007). The effect of ϕ is smaller but also not negligible.

CONCLUSION

A seismic deterministic and probabilistic analysis of a footing resting at the top of a slope is analyzed using limit analysis models and finite element numerical simulations. The major conclusion of the paper can be summarized as follows: In the deterministic analysis, good agreement is found between the limit analysis and Plaxis results with or without seismic loading. Also, for a given case, when the slope height H is small and d/B increases to reach a limit value, the failure tends to be developed under the footing only without passing by the slope. However, when H is high and d/B increases, the most critical failure was found to be the one passing by the slope only. In the reliability analysis, the same limit for d/B obtained using the deterministic analysis is obtained. The variation of c , ϕ , and P_s affect highly the reliability index and have approximately the same degree of influence on the slope-footing system reliability obtained by the M1 mechanism.

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