Influence of soil spatial variability and stochastic Ground-Motion on the dynamic behavior of a slope

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ABSTRACT

A probabilistic dynamic approach is used for the slope stability analysis. In this approach, the effect of both the soil spatial variability and the variability of the Ground-Motion (GM) time history on the dynamic responses (amplification, permanent displacement) are studied and discussed. The soil shear modulus G is considered as an isotropic non-Gaussian random field. The simulation of random acceleration time histories based on a real target accelerogram is done using a fully nonstationary stochastic model in both the time and the frequency domains. The deterministic model is based on numerical simulations. An efficient uncertainty propagation methodology which builds up a sparse polynomial chaos expansion for the dynamic responses is used. The probabilistic numerical results have shown that: (i) the decrease in the autocorrelation distance of the shear modulus leads to a small variability of the dynamic responses; (ii) the randomness of the earthquake GM has a significant influence on the variability of the dynamic responses; (iii) the probabilistic mean values of the dynamic responses are more critical than the deterministic ones.

INTRODUCTION

The seismic stability of slopes is widely investigated in literature using deterministic approaches. However, the material properties of soils are known to vary greatly from point to another. Things are more complicated when dealing with dynamic loading situations where the seismic excitation is uncertain. In this paper, the effects of both the soil spatial variability and the variability of the Ground-Motion (GM) on the dynamic responses of a simple slope are studied. Few authors have worked on the analysis of the dynamic soil behavior using probabilistic approaches [e.g. Koutsourelakis et al. (2002), Popescu et al. (2006)]. In these works, two main deficiencies can be detected: Firstly, the classical Monte Carlo Simulation (MCS) methodology was used by Koutsourelakis et al. (2002) by employing a very small number of realizations (e.g. 50 simulations). Second, the stochastic model used by Koutsourelakis et al. (2002) to model the variability of the GM was based on the spectral representation in order to simulate accelerograms which are compatible with a prescribed response spectrum and not a real GM acceleration.

In this study, the two mentioned deficiencies will be avoided by (i) using a more efficient probabilistic approach which is the Sparse Polynomial Chaos Expansion (SPCE) [Blatman and Sudret (2010), Al-Bittar and Soubra (2011)]; (ii) simulating a random acceleration time history based on a real target accelerogram using a fully nonstationary stochastic model in both the time and the frequency domains [cf. Rezaeian and Der Kiureghian (2008, 2010)].

The deterministic model is based on numerical simulations using the dynamic option of the finite difference code FLAC^{3D}. The Expansion Optimal Linear Estimation (EOLE) methodology proposed by Li and Der Kiureghian (1993) is used to generate the random field of the shear modulus G. Samples of the synthetic GM time-histories were generated and a dynamic stochastic calculation for each realization was performed to compute the dynamic responses (i.e. the permanent displacement at the toe of the slope and the maximum amplification of the acceleration at the top of the slope). The paper is organized as follows: The first three sections aim at presenting (i) the method used to generate the random field of the shear modulus *G*, (ii) the method used to generate the stochastic synthetic accelerograms based on a real target one and finally (iii) the SPCE methodology employed to determine the analytical expression of the dynamic system responses. These sections are followed by a presentation of the probabilistic numerical results in which only the soil spatial variability is first considered and then combined with the time variability of the GM in order to highlight its effect on the variability of the dynamic responses.

GENERATION OF A NON-GAUSSIAN RANDOM FIELD

The soil shear modulus (*G*) is modeled herein as a non-Gaussian '*NG*' (lognormal) random field $Z_G^{NG}(x, y)$. It is described by: (i) constant mean μ_G and standard deviation σ_G, (ii) non-Gaussian marginal cumulative distribution function F_G, and (iii) a square exponential autocorrelation function ρ_Z^{NG} [(x, y), (x', y')] which gives the values of the correlation function between two arbitrary points *(x, y)* and *(x', y')*. This autocorrelation function is given as follows:

$$\rho_z^{NG}[(x, y), (x', y')] = \exp\left(-\left(\frac{x-x'}{a_x}\right)^2 - \left(\frac{y-y'}{a_y}\right)^2\right) \quad (1)$$

where a_x and a_y are the autocorrelation distances along *x* and *y* respectively. The EOLE method proposed by Li and Der Kiureghian (1993) is used herein to generate the random field of *G*. In this method, one should first define a stochastic grid composed of *q* grid points (or nodes) obtained from the different combinations of *H* points in the *x* (or horizontal) direction, and *V* points in the *y* (or vertical) direction. The grid points are assembled in a vector $Q=\{Q_n=(x_h, y_v)\}$ where $h=1, \ldots, H$, $v=1, \ldots, V$, $n=1, \ldots, q$ and $q=HxV$. The values of the field at the different grid points are assembled in a vector $\chi = \{ \chi_n = Z(x_h, y_v) \}$. The correlation $\left(\Sigma^{NG}_{\chi;\chi} \right)_{i,j}$ between two arbitrary points Q_i, Q_j is calculated using Equation (1) as follows:

$$\left(\Sigma^{NG}_{\chi;\chi} \right)_{i,j} = \rho^{NG}_{z} \left[Q_i, Q_j \right] \tag{2}$$

where $i=1, \ldots, q$ and $j=1, \ldots, q$. Notice that the matrix $\Sigma^{NG}_{\chi;\chi}$ in equation (2) provides the correlation between each point in the vector χ and all the other points of the same vector. The non-Gaussian autocorrelation matrix $\Sigma^{NG}_{\chi;\chi}$ should be transformed into the Gaussian space using the Nataf transformation. As a result, one obtains a Gaussian autocorrelation matrix $\Sigma^{G}_{\chi;\chi}$ that can be used to discretize the random field of the shear modulus G as follows:

$$\tilde{Z}_G(x, y) = \mu_G + \sigma_G \sum_{j=1}^{N} \frac{\xi_j}{\sqrt{\lambda_j}} .\phi_j^T .\Sigma_{Z(x,y);\chi} \tag{3}$$

where (λ_j, ϕ_j) are the eigenvalues and eigenvectors of the Gaussian autocorrelation matrix $\Sigma^{G}_{\chi;\chi}$, $\Sigma_{Z(x,y);\chi}$ is the correlation vector between each point in the vector χ and the value of the field at an arbitrary point (x, y), ξ_j is a standard normal random variable, and N is the number of terms (expansion order) retained in EOLE method. In Equation (3), one obtains the solution of a Gaussian random field. The extension to the case of a lognormal field is performed as follows:

$$\tilde{Z}^{NG}_G(x, y) = Exp\left(\tilde{Z}_G(x, y) \right) \tag{4}$$

GENERATION OF STOCHASTIC GROUND MOTION ACCELEROGRAMS

In this paper, the method proposed by Rezaeian and Der Kiureghian (2010) was used to generate stochastic acceleration time histories from a target accelerogram. This method consists in fitting a parameterized stochastic model that is based on a modulated, filtered white-noise process to a recorded ground motion. The parameterized stochastic model in its continuous form is defined as:

$$x(t) = q(t, \alpha) \left\{ \frac{1}{\sigma_h(t)} \left[\int_{-\infty}^{t} h\left[t-\tau, \lambda(\tau) \right] w(\tau) d\tau \right] \right\} \tag{5}$$

In this expression, $q(t, \alpha)$ is a deterministic, positive, time-modulating function with parameters α_i controlling its shape and intensity; $w(\tau)$ is a white-noise process; the integral inside the brackets is a filtered white-noise process with $h\left[t-\tau, \lambda(\tau) \right]$ denoting the Impulse-Response Function (IRF) of the filter where $\lambda(\tau)$ is a time-varying vector of parameters; and $\sigma_h^2(t) = \int_{-\infty}^{t} h^2 \left[t-\tau, \lambda(\tau) \right] d\tau$ is the variance of the integral process. Because of the normalization by $\sigma_h(t)$, the process inside the brackets has unit variance. As a result, $q(t, \alpha)$ equals the standard deviation of the resulting process x(t). It should be noticed that the modulating function $q(t, \alpha)$ completely defines the temporal characteristics of the process, whereas the form of the filter IRF and its time-varying parameters define the spectral characteristics of the process. In this study, a 'Gamma' modulating function is used as follows:

$$q(t,\alpha)=\alpha_1 t^{\alpha_2-1}\exp(-\alpha_3 t) \tag{6}$$

In this equation, $\alpha=(\alpha_1,\alpha_2,\alpha_3)$ where $(\alpha_1,\alpha_3)>0$, and $\alpha_2>1$. Of the three parameters, α_1 controls the intensity of the process, α_2 controls the shape of the modulating function and α_3 controls the duration of the motion. These parameters are related to three physically based parameters $(\bar{I}_a, D_{5-95}, t_{mid})$ which describe the real recorded GM in the time domain; where $\bar{I}_a$ is the Arias Intensity (AI) and D_{5-95} represents the effective duration of the motion. D_{5-95} is defined as the time interval between the instants at which the 5% and 95% of the expected AI are reached respectively. Finally, t_{mid} is the time at the middle of the strong-shaking phase. It is selected as the time at which 45% of the expected AI is reached. The relations between $\alpha=(\alpha_1,\alpha_2,\alpha_3)$ and $(\bar{I}_a, D_{5-95}, t_{mid})$ are given in details in Rezaeian and Der Kiureghian (2010).

For the filter IRF, we select a form that corresponds to the pseudo-acceleration response of a single-degree-of-freedom linear oscillator:

$$\begin{aligned} h\left[t-\tau,\lambda(\tau)\right] &= \frac{\omega_f(\tau)}{\sqrt{1-\zeta_f^2(\tau)}}\exp\left[-\zeta_f(\tau)\omega_f(\tau)(t-\tau)\right]\times\sin\left[\omega_f(\tau)\sqrt{1-\zeta_f^2(\tau)}(t-\tau)\right] \quad t\le\tau \\ &= 0 \quad \text{otherwise} \end{aligned} \tag{7}$$

where $\lambda(\tau)=(\omega_f(\tau),\zeta_f(\tau))$ is the set of time-varying parameters of the IRF with $\omega_f(\tau)$ denoting the frequency of the filter and $\zeta_f(\tau)$ denoting its damping ratio. These two parameters are related to two physical parameters that describe the recorded GM in the frequency domain and which are respectively the predominant frequency and the bandwidth of the GM. For more details about the identification procedure between the recorded GM and the stochastic model described previously, the reader may refer to Rezaeian and Der Kiureghian (2008, 2010).

SPARSE POLYNOMIAL CHAOS EXPANSION (SPCE) METHODOLOGY

The polynomial chaos expansion (PCE) methodology aims at replacing a computationally-expensive deterministic model whose input parameters are modeled by random variables by a PCE (called meta-model) which is an approximate analytical formula. This allows one to calculate the system response using an analytical equation with no time cost [Blatman and Sudret (2010)]. The coefficients of the PCE are computed herein using a regression approach.

For a deterministic numerical model with M input uncertain parameters, the uncertain parameters should first be represented by independent standard normal random variables $\{\xi_i\}_{i=1,\ldots,M}$ gathered in a random vector ξ. The random response Γ of our mechanical model can then be expressed by a PCE of order p fixed by the user as follows:

$$\Gamma_{PCE}(\xi) = \sum_{\beta=0}^{\infty} a_\beta \Psi_\beta(\xi) \cong \sum_{\beta=0}^{P-1} a_\beta \Psi_\beta(\xi) \tag{8}$$

where P is the number of terms retained in the truncation scheme, a_β are the unknown PCE coefficients to be computed and Ψ_β are multivariate (or multidimensional) Hermite polynomials which are orthogonal with respect to the joint probability distribution function of the standard normal random vector ξ. These multivariate polynomials are given by $\Psi_\beta = \prod_{i=1}^{M} H_{\alpha_i}(\xi)$, where $H_{\alpha_i}(.)$ is the α_i-th one-dimensional Hermite polynomial and α_i are a sequence of M non-negative integers $\{\alpha_1,...,\alpha_M\}$. In practice, one should truncate the PCE representation by retaining only the multivariate polynomials of degree less than or equal to the PCE order p. For this reason, a classical truncation scheme based on the first order norm is generally adopted in literature. This first order norm is defined as follows: $\|\alpha\|_1 = \sum_{i=1}^{M} \alpha_i$. The classical truncation scheme suggests that the first order norm should be less than or equal to the order p of the PCE. Using this method of truncation, the number P of the unknown PCE coefficients is given by $P = \frac{(M+p)!}{M!\,p!}$. Thus, the number P of the PCE coefficients dramatically increases with the number M of the random variables and the order p of the PCE. To overcome such a problem, a new truncation strategy called 'hyperbolic truncation scheme' based on a so-called q-norm was proposed by Blatman and Sudret (2010). The q-norm is given by $\|\alpha\|_q = \left(\sum_{i=1}^{M} \alpha_i^q\right)^{1/q}$ where q is a coefficient ($0<q<1$). The hyperbolic truncation scheme suggests that the q-norm should be less than or equal to the order p of the PCE. This strategy is based on the fact that the multidimensional polynomials Ψ_β corresponding to high-order interaction are associated with very small values for the coefficients a_β. The proposed methodology leads to a SPCE that contains a small number of unknown coefficients which can be calculated from a reduced number of calls of the deterministic model. This is of particular interest in the present case of a random field which involves a significant number of random variables. This strategy will be used in this paper to build up a SPCE of the system response using an iterative procedure [Blatman and Sudret (2010)]. Once the unknown coefficients of the SPCE are determined, the PDF of the dynamic responses can be computed using Monte Carlo simulation technique on the meta-model.

NUMERICAL RESULTS

The aim of this section is to present the probabilistic results. It should be remembered here that the dynamic system responses involve the permanent displacement at the toe (computed as the difference between the displacements at the toe and at the bottom of the soil mass) and the maximum amplification of the acceleration at the top of the slope (computed as the ratio between the maximal acceleration at the top of the slope and that at the bottom of the soil mass). In this study, the effect of both the soil spatial variability and the variability of the Ground-Motion (GM) on the dynamic responses is considered. The soil shear modulus G is considered as an isotropic (i.e. a_x=a_y) lognormal random field. The mean value and the coefficient of variation of G are respectively $\mu_G = 112.5MPa$ and $Cov_G = 40\%$. In order to simulate the stochastic synthetic time histories, the Kocaeli (Turkey 1999) earthquake is used as the target accelerogram (see Fig.1). The deterministic model is based on numerical simulations using the dynamic option of the finite difference code FLAC^{3D}. The slope geometry considered in the analysis is shown in Fig.2. It should be noted that the size of a given element in the mesh depends on both the autocorrelation distance of the soil property (shear modulus) and the wavelength λ associated with the highest frequency component f_{max} of the input signal. For the autocorrelation distance of the soil property, Der Kiureghian and Ke (1988) have suggested that the length of the smallest element in a given direction (horizontal or vertical) should not exceed 0.5 times the autocorrelation distance in that direction. As for the wavelength λ associated with the highest frequency component f_{max} of the input signal, FLAC manual has suggested that the largest element should not exceed 1/10 to 1/8 this wavelength λ in order to avoid the distortion that may occur for the propagating waves. A maximal size element of 2m was used to respect the two above conditions. For the boundary conditions, the bottom horizontal boundary was subjected to an earthquake acceleration signal and free field boundaries were applied to the right and left vertical boundaries. The numerical simulations are performed using an elastic perfectly-plastic soil model based on Mohr-Coulomb failure criterion. The values of the bulk modulus K, the cohesion c, the friction angle φ, the dilation angle ψ, and the soil unit weight γ are as follows: K=133MPa, c=10kPa, φ=30°, ψ =20°, and γ =18kN/m^3.

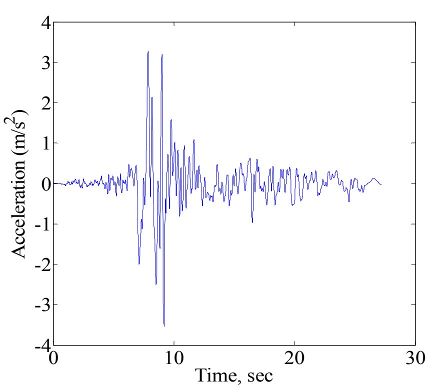

Figure 1. Kocaeli (Turkey 1999) accelerogram

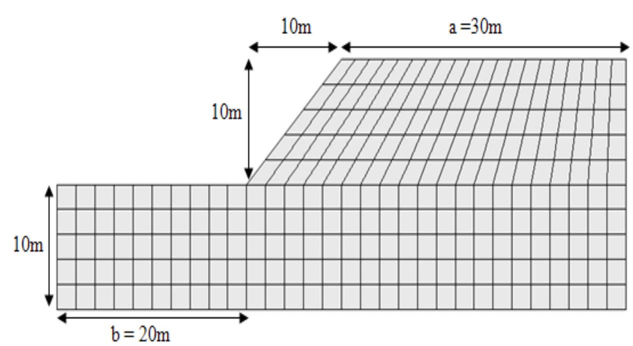

Figure 2. Slope geometry and mesh used in the analysis

In the following sections, one examines the effect of the soil spatial variability on both the amplification at the top of the slope and the permanent displacement at the toe of this slope in both cases of deterministic and stochastic GM accelerograms.

Effect of the soil spatial variability on the amplification at the top of the slope

The effect of the soil spatial variability on the amplification at the top of the slope in both cases of deterministic and stochastic GM accelerograms is studied and presented in Figs. 3, 4 and Table 1. In the present paper, the isotropic autocorrelation distance a_x=a_y was non-dimensionalized by dividing it by the height of the slope H (θ=a_x/H=a_y/H). Different values of the isotropic autocorrelation distance (θ=0.5, 1, 2, 3, 5) were considered in the analyses. Figs. 3 and 4 show that the PDF is less spread out when the isotropic autocorrelation distance θ decreases, i.e., the variability of the amplification at the top of the slope decreases with the increase in the soil heterogeneity. This can be explained by the fact that for small values of the autocorrelation distance, the fluctuations of the shear modulus are averaged to a mean value. This mean is close to the probabilistic mean value of the random field G. This leads to close values of the amplification and thus to a smaller variability in this response. The comparison between Figs. 3 and 4 (see also Table 1) shows that the randomness of the earthquake GM has a significant effect on the variability of the amplification. Table 1 shows that for the range of the autocorrelation distances considered in this study, the coefficient of variation COV of the amplification is between 2.78% and 10.91% when deterministic GM accelerogram is used. This range of COV significantly increases when the randomness of the earthquake GM is introduced. In this case, the COV of the amplification varies between 4.24% and 31.78%. One can notice that for the largest autocorrelation distance θ=5, the variability of the amplification in the case where a stochastic GM accelerogram was used is 2.9 time larger than the one obtained with the deterministic GM accelerogram.

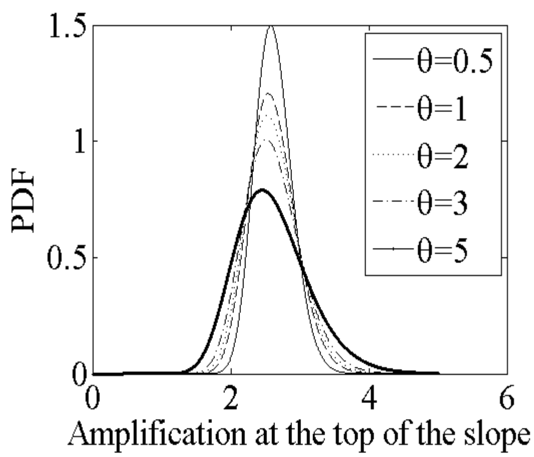

Figure 3. Amplification at the top of the slope with deterministic GM

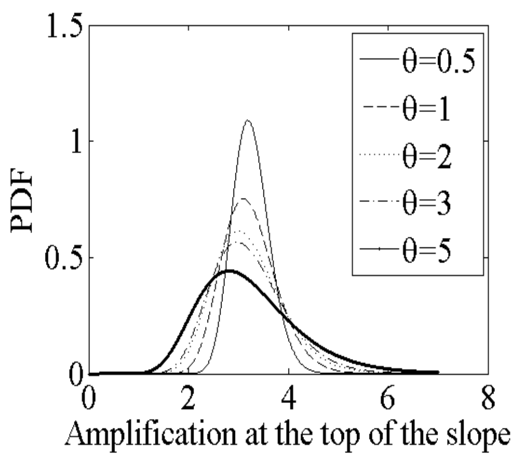

Figure 4. Amplification at the top of the slope with stochastic GM

Table 1 also shows that the autocorrelation distance θ has practically no effect on the probabilistic mean value of the amplification. This mean value was found to be larger than the corresponding deterministic value. This means that the probabilistic results are much more critical than the deterministic value with a difference of 5% in the case where a deterministic GM accelerogram is used, and 29% in the case where a stochastic GM accelerogram is used.

Table 1. Effect of the autocorrelation distance θ on the statistical moments (μ, σ) **of the amplification**

	θ	Mean μ	Standard deviation σ	COV (%)	Deterministic amplification
Deterministic GM	0.5	2.6	0.07	2.78	2.48
	1	2.6	0.11	4.36	
	2	2.6	0.13	5.18	
	3	2.6	0.17	6.36	
	5	2.6	0.28	10.91	
	θ	Mean μ	Standard deviation σ	COV (%)	Deterministic amplification
Stochastic GM	0.5	3.2	0.14	4.24	2.48
	1	3.2	0.30	9.30	
	2	3.2	0.47	14.61	
	3	3.2	0.57	17.56	
	5	3.2	1.03	31.78	

Effect of the soil spatial variability on the permanent displacement at the toe of the slope

The effect of the soil spatial variability on the permanent displacement at the toe of the slope for both cases of deterministic and stochastic GM accelerograms is studied and presented in Figs. 5, 6 and Table 2. The same values of the isotropic autocorrelation distance θ used in the previous section are also used herein. Fig. 5 shows that the PDFs are very close to each other and thus the shear modulus variability has a small influence on the permanent displacement. This is because the permanent displacement appears only when the plastic phase is reached which means that the effect of the shear modulus G on this response is relatively small. Table 2 confirms this observation because very small values of the COV of the permanent displacement are obtained when only the spatial variability of G is considered. On the other hand, the comparison between the results of Figs. 5 and 6 (see the corresponding statistical moments in Table 2) shows that the randomness of the earthquake GM considerably affects the permanent displacement. High values of the COV are detected because of the important increase in the mean value of the permanent displacement due to the variability of the GM. Table 2 also shows that the mean value of the permanent displacement presents a maximum. This maximum was detected when θ=2, i.e. when the isotropic autocorrelation distance is equal to the height of the soil domain. When θ decreases from 5 to 2, one can notice that the mean of the permanent displacement increases. This can be explained by the fact that increasing the soil heterogeneity introduces weak zones with small values of the shear modulus G, thus leading to larger values of the permanent displacement. The decrease in the permanent displacement for values of θ smaller than 2 may be explained by the fact that as the autocorrelation distance decreases, the propagating wave can face some stiff zones which reduce the permanent displacement.

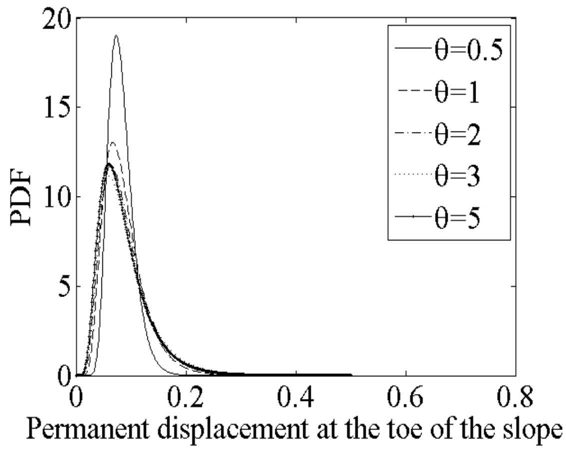

Figure 5. Permanent displacement at the toe of the slope with deterministic GM

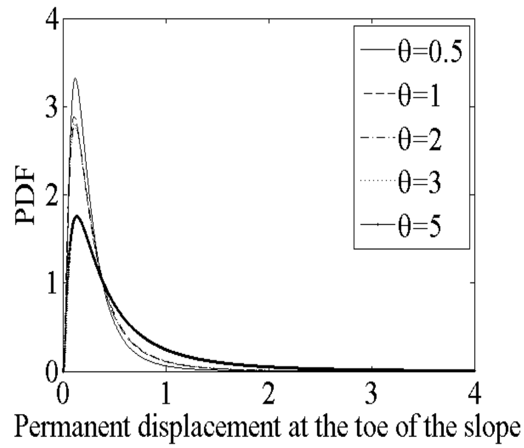

Figure 6. Permanent displacement at the toe of the slope with stochastic GM

Finally, on can notice also that introducing the soil spatial variability and the randomness of GM lead to more critical results since all the mean values of the permanent displacement obtained in the probabilistic study are larger than the corresponding deterministic value.

Table 2. Effect of the autocorrelation distance θ on the statistical moments (μ, σ) **of the permanent displacement**

	θ	Mean μ [m]	Standard deviation σ [m]	COV (%)	Deterministic permanent displacement
Deterministic GM	0.5	82.0×10^{-3}	0.5×10^{-3}	0.61	40.7×10^{-3}
	1	86.2×10^{-3}	1.4×10^{-3}	1.62	
	2	88.4×10^{-3}	2.0×10^{-3}	2.26	
	3	87.5×10^{-3}	2.1×10^{-3}	2.40	
	5	85.5×10^{-3}	2.5×10^{-3}	2.92	
	θ	Mean μ [m]	Standard deviation σ [m]	COV (%)	Deterministic permanent displacement
Stochastic GM	0.5	262.0×10^{-3}	59.6×10^{-3}	22.75	40.7×10^{-3}
	1	264.6×10^{-3}	124.8×10^{-3}	47.16	
	2	274.0×10^{-3}	126.7×10^{-3}	46.24	
	3	271.7×10^{-3}	135.9×10^{-3}	50.02	
	5	255.7×10^{-3}	279.3×10^{-3}	109.23	

CONCLUSIONS

The effect of both the soil spatial variability and the variability of the Ground-Motion (GM) on the dynamic responses (amplification, permanent displacement) is studied. The soil shear modulus G is considered as an isotropic lognormal random field. The simulation of random acceleration time histories based on a real target accelerogram is done using a fully nonstationary stochastic model in both the time and the frequency domains. The deterministic model was based on numerical simulations. The probabilistic methodology adopted in this paper makes use of a non-intrusive approach to build up a sparse polynomial chaos expansion (SPCE) for the dynamic system responses. The main conclusions can be summarized as follows: (i) the decrease in the autocorrelation distance of the shear modulus leads to a decrease in the variability of the dynamic responses; this decrease being more significant for the amplification; (ii) the randomness of the earthquake GM has a significant effect on the variability of the dynamic responses; (iii) the isotropic autocorrelation distance affects the probabilistic mean values of plastic responses (i.e. the permanent displacement); its effect being negligible on the elastic responses (i.e. the amplification).

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