

Patrick Launeau

Quantitative Image Analysis of Minerals and Rocks

Fabric analysis

- Shape preferred orientation (SPO) vs. strain quantification.
- Intercepts in digital images : a tool to analyze interconnection of grains in rocks vs. inertia tensor of individualized grains
- SPO vs Spatial distribution (Fry)
- Ellipsoid of SPO and strain by combining 3 \perp images.



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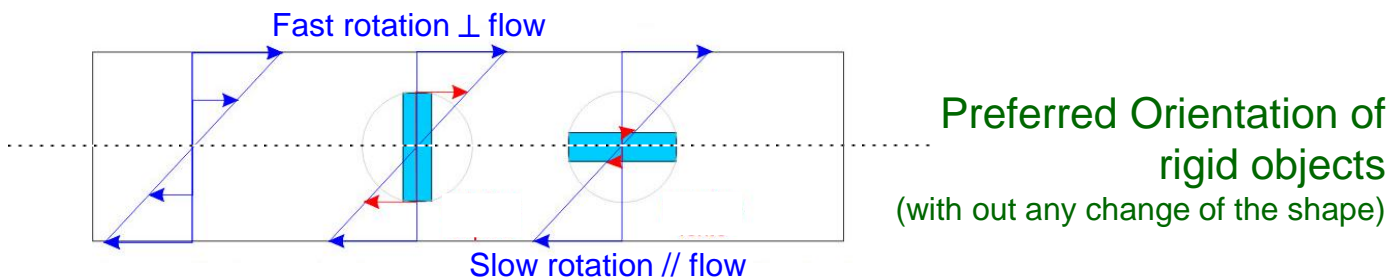
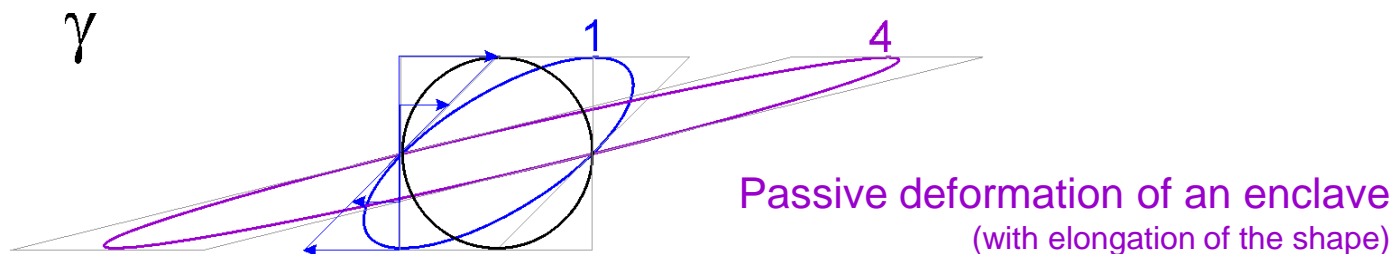
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Fabric analysis

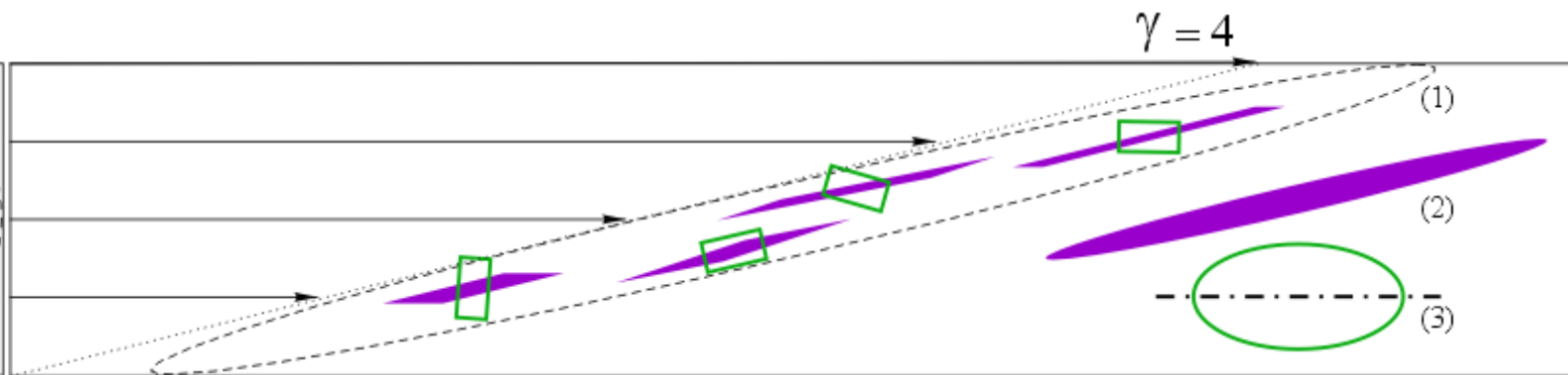
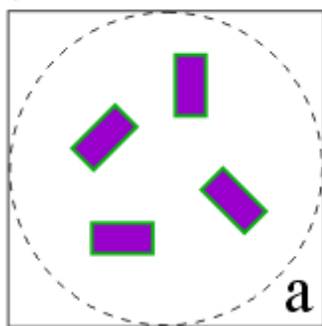
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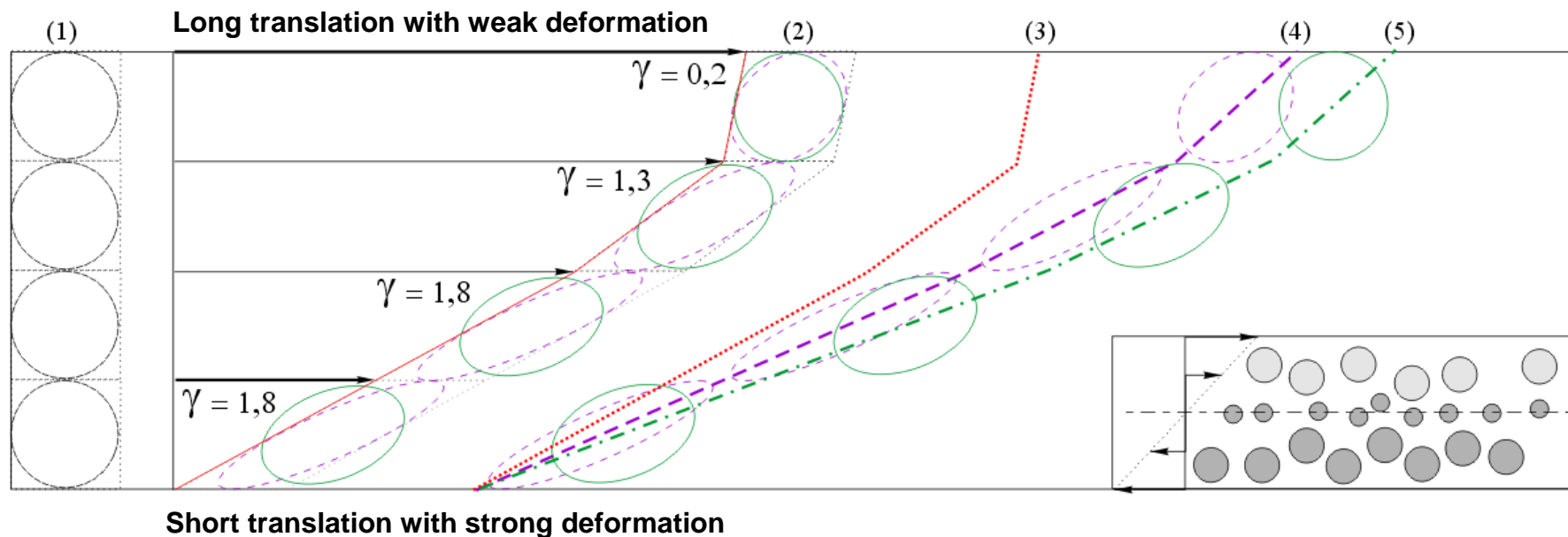
Passive / active deformation



$\gamma = 0$



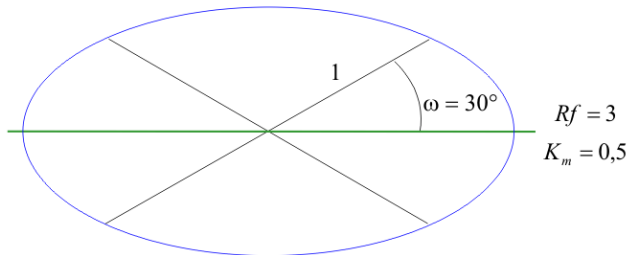
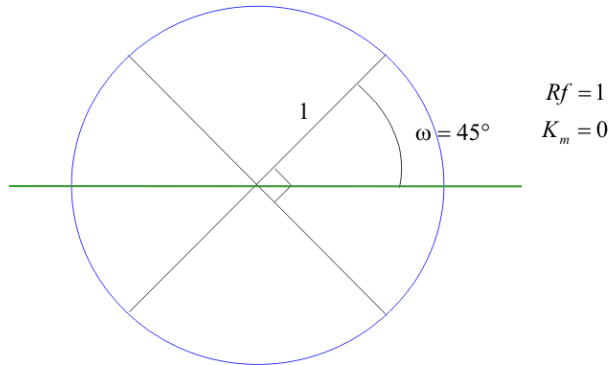
In a more realistic magma flow ...



- (1) Initial position of a magma bubble
- (2) Final position of its vertical section in red, its shape in purple, the preferred orientation of its microlithes in green
- (3) Alignment of the initial vertical sections of the magma bubbles
- (4) Alignment of the ellipses of magma deformed passively
- (5) Alignment of the ellipses of crystal preferred orientation

Preferred Orientation (PO)

Cosines directions (Harvey & Laxton, 1980)



$\omega = 0$ $Rf = \infty$
 $K_m = 1$

$$\mathbf{M} = \frac{1}{N} \begin{bmatrix} \sum \cos^2 \varphi_i & \sum \cos \varphi_i \sin \varphi_i \\ \sum \sin \varphi_i \cos \varphi_i & \sum \sin^2 \varphi_i \end{bmatrix}$$

$$\mathbf{M} = \begin{bmatrix} \cos \varphi & \sin \varphi \\ -\sin \varphi & \cos \varphi \end{bmatrix} \cdot \begin{bmatrix} \sqrt{Rf} & 0 \\ 0 & 1/\sqrt{Rf} \end{bmatrix} \cdot \begin{bmatrix} \cos \varphi & -\sin \varphi \\ \sin \varphi & \cos \varphi \end{bmatrix}$$

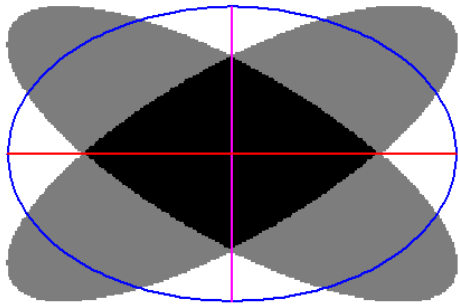
$$Rf = \frac{\cos^2 \omega}{\sin^2 \omega} = \frac{1}{\tan^2 \omega} \quad K_m = \frac{Rf - 1}{Rf + 1}$$

Shape Preferred Orientation (SPO)

SPO → of shapes with long and short axes

A n=2 a=5,9926 cm b=3,9318 cm F=1,524 [1,729]n [1,734]b . 89,99°
K=0,398, Kn=0,498 (0,799), Kb=0,501 (0,795)

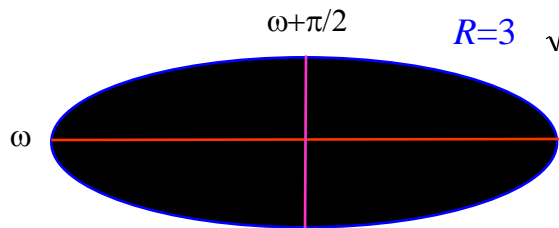
$$R=1,5 \quad \sqrt{Rf} = 1,7$$



$$R \leq \sqrt{Rf} \quad \text{when } r \geq 10$$

$$R < \sqrt{Rf} \quad \text{when } 1 < r < 10$$

$$R=3 \quad \sqrt{Rf} = \infty$$



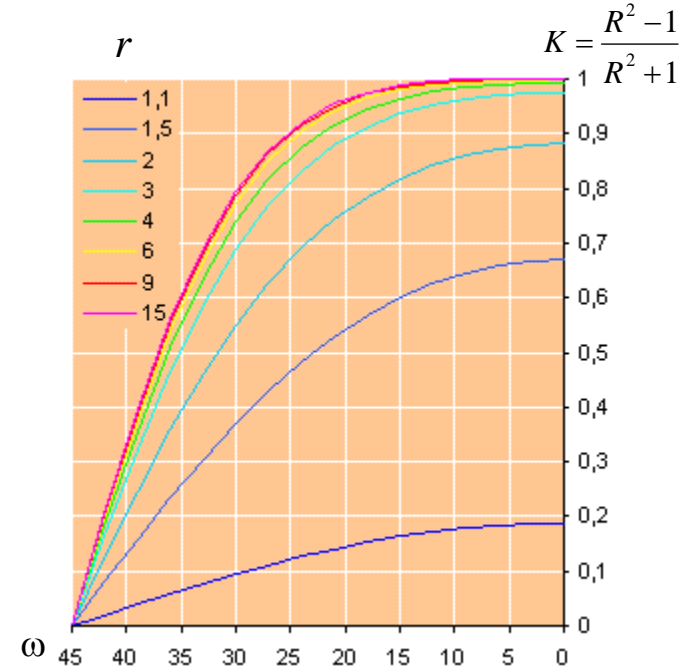
$$R = \frac{d(\omega)}{d(\omega + \pi/2)}$$

Normalization to k

$$k = \frac{r^2 - 1}{r^2 + 1}$$

$$K_n = K / k$$

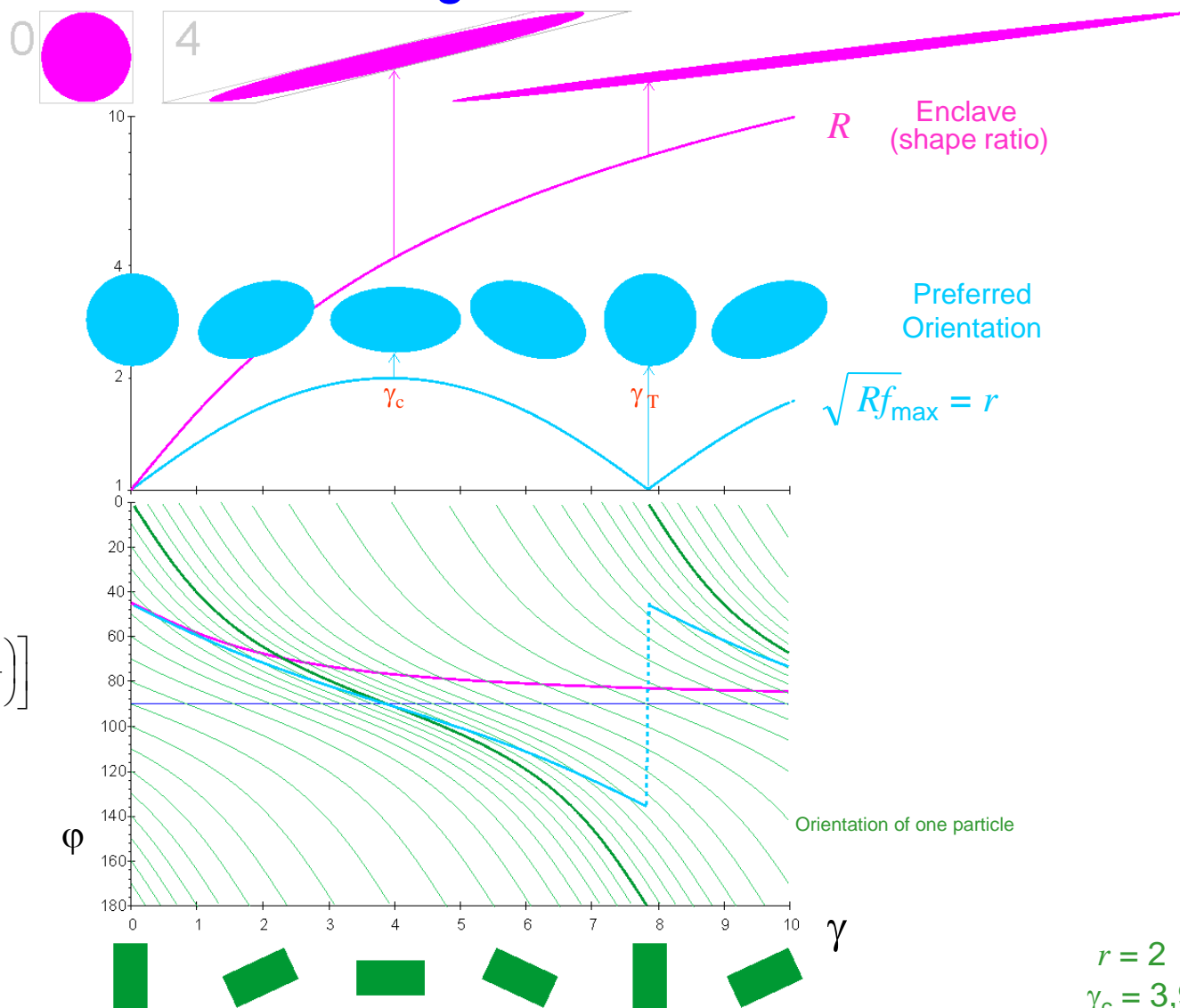
$$R_n = \sqrt{\frac{1 + K_n}{1 - K_n}}$$



2D magma flow

SPO with strain record
(passive deformation)

SPO without strain record
(active deformation)



Jeffery 1922

$$\tan \beta' = r \cdot \tan \left[\frac{r \cdot \gamma}{r^2 + 1} + \arctan \left(\frac{\tan \beta}{r} \right) \right]$$

$$\gamma_c = \frac{\pi}{\sqrt{1 - k^2}} \quad k = \frac{r^2 - 1}{r^2 + 1}$$

$r = 2$
 $\gamma_c = 3,93$

Simple shear in 3D

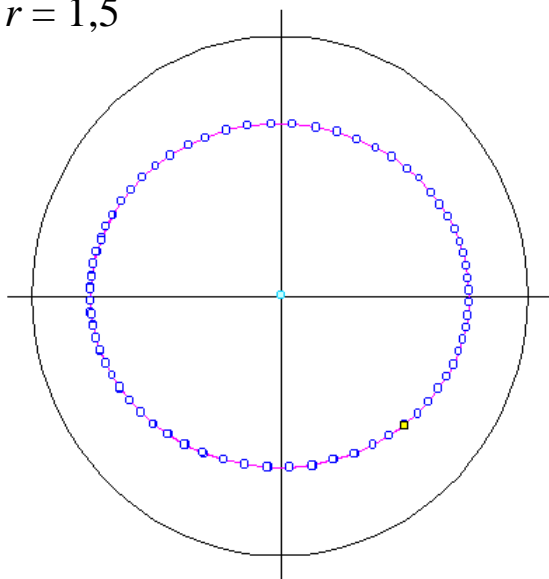
3D

Jeffery (1922), Reed and Tryggvason (1974) et Willis (1977)

$$\tan \beta' = r \cdot \tan \left[\frac{r \cdot \gamma}{r^2 + 1} + \arctan \left(\frac{\tan \beta}{r} \right) \right]$$

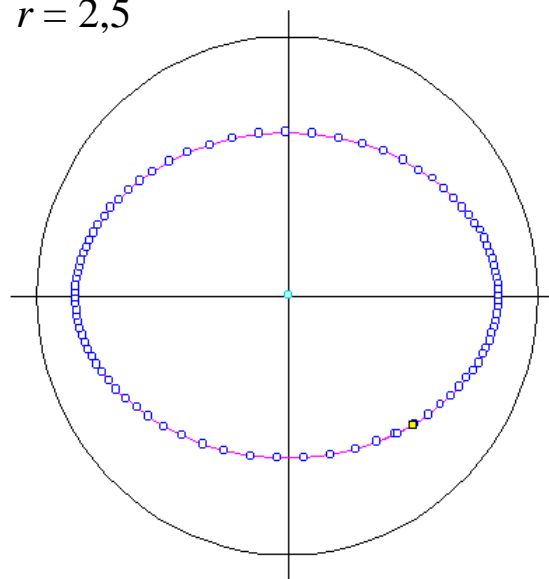
$$\tan^2 \psi' = \tan^2 \psi \cdot \left(\frac{r \cdot \cos^2 \beta + \sin^2 \beta}{r \cdot \cos^2 \beta' + \sin^2 \beta'} \right)$$

$r = 1,5$

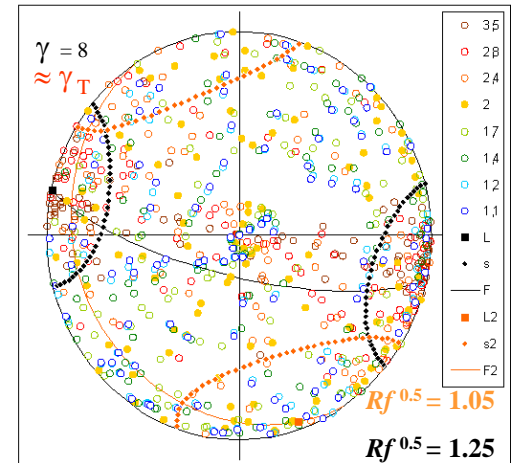
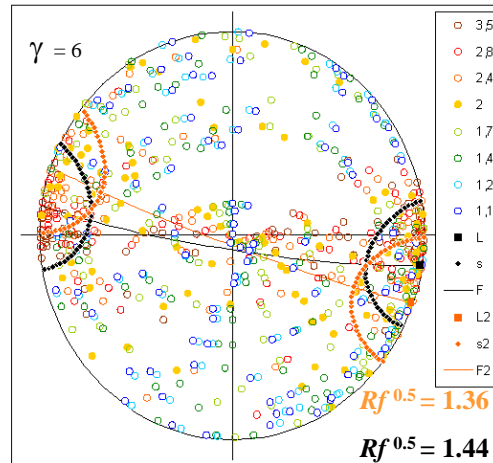
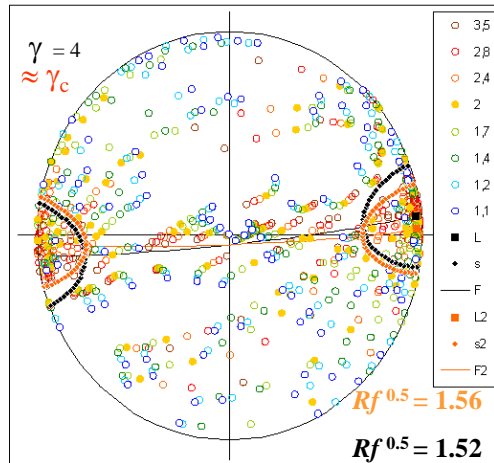
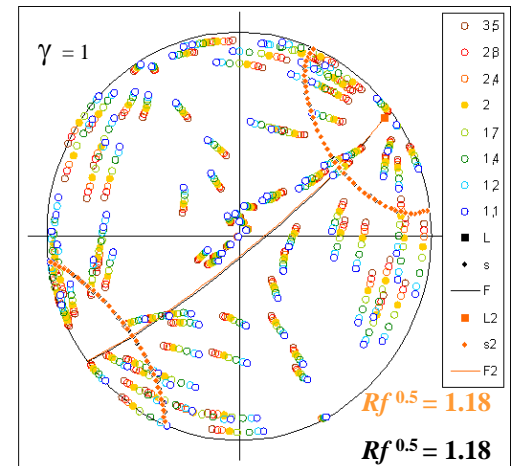
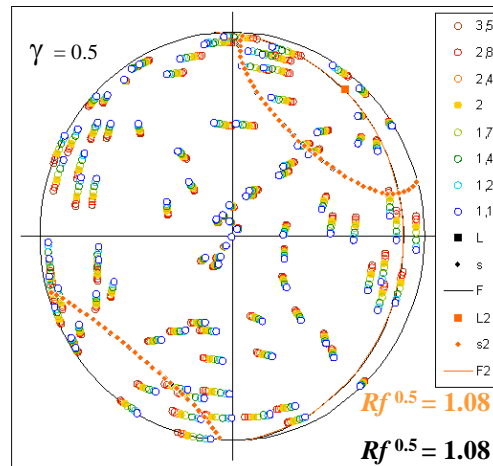
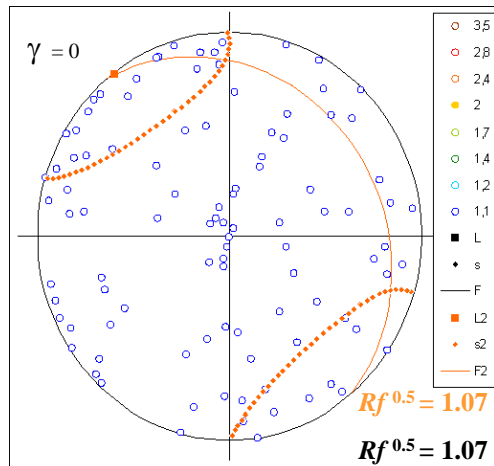


1,5 : 1 : 1

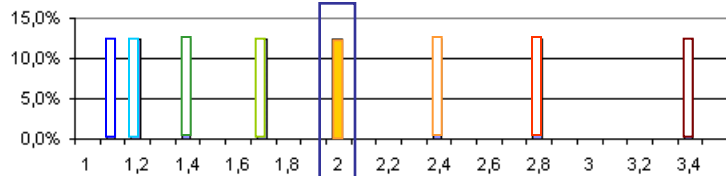
$r = 2,5$



2,5 : 1 : 1



Ellipsoid 2:1:1 $r_{a/c} = 2$



The PO of one class of aspect ratio is cyclic
The PO of standard CSD is not cyclic

Jeffery (1922), Reed and Tryggvason (1974) et Willis (1977)

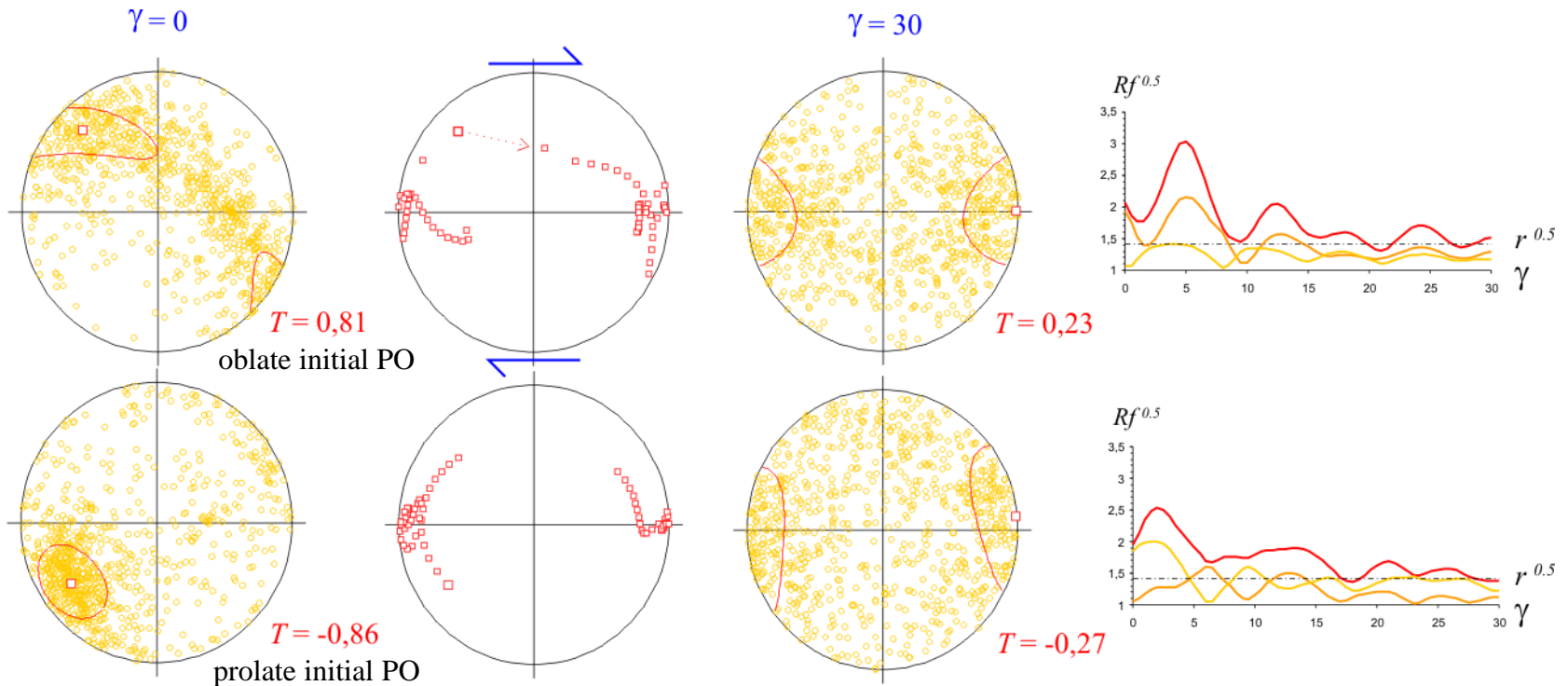
$$\tan \beta' = r \cdot \tan \left[\frac{r \cdot \gamma}{r^2 + 1} + \arctan \left(\frac{\tan \beta}{r} \right) \right]$$

$$\tan^2 \psi' = \tan^2 \psi \cdot \left(\frac{r \cdot \cos^2 \beta + \sin^2 \beta}{r \cdot \cos^2 \beta' + \sin^2 \beta'} \right)$$

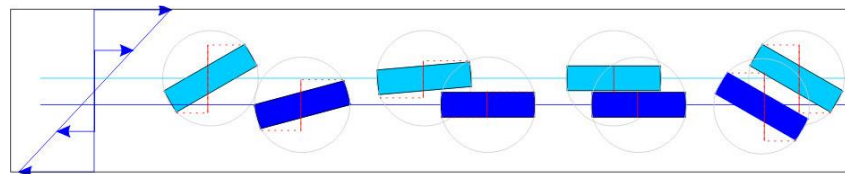
Ellipsoid 2:1:1

$r_{a/c} = 2$

SPO with strong initial SPO tend toward a weak SPO along a magma flow



Passive / active deformation



Particle interactions
may reduce the object
rotation

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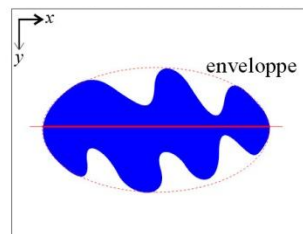


Image analysis

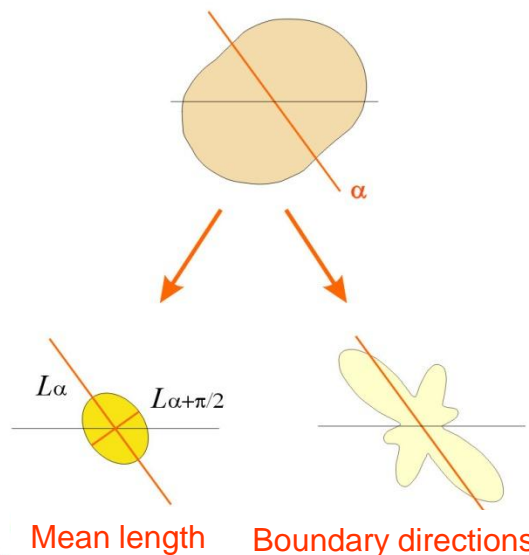
First hypothesis :

do we have
isolated crystals
or aggregates ?

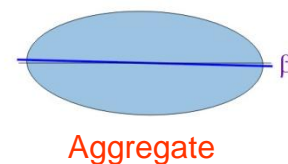
Aggregate



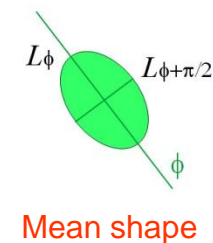
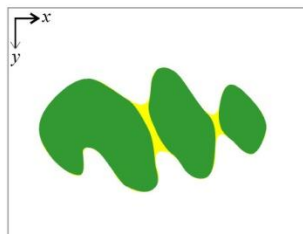
Intercepts method



Inertia tensor method

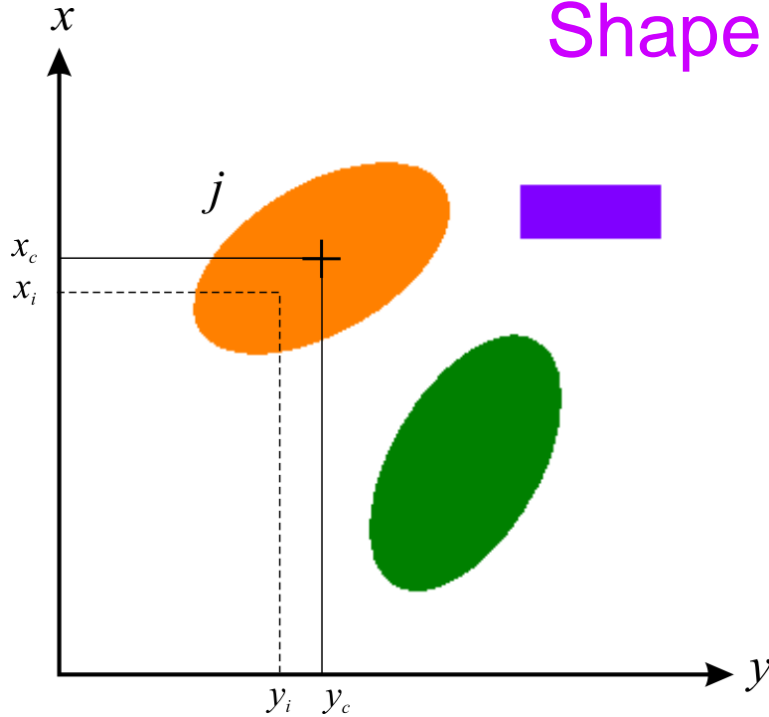


Grain segmentation



Shape analysis with inertia tensor

Case of 1 object



$$x_c = \frac{1}{A} \sum_i x_i$$

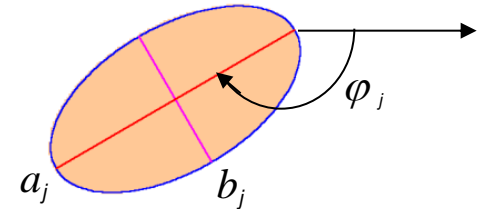
$$y_c = \frac{1}{A} \sum_i y_i$$

$$m_{xxj} = \frac{1}{A} \sum_i (x_i - x_c)^2$$

$$m_{xyj} = \frac{1}{A} \sum_i (x_i - x_c)(y_i - y_c)$$

$$m_{yyj} = \frac{1}{A} \sum_i (y_i - y_c)^2$$

$$\mathbf{M}_j = \begin{vmatrix} m_{xxj} & m_{xyj} \\ m_{xyj} & m_{yyj} \end{vmatrix}$$



$$r_j = \frac{a_j}{b_j}$$

$$\mathbf{M}_j = \begin{bmatrix} \cos \varphi_j & \sin \varphi_j \\ -\sin \varphi_j & \cos \varphi_j \end{bmatrix} \cdot \begin{bmatrix} a_j^2/4 & 0 \\ 0 & b_j^2/4 \end{bmatrix} \cdot \begin{bmatrix} \cos \varphi_j & -\sin \varphi_j \\ \sin \varphi_j & \cos \varphi_j \end{bmatrix}$$

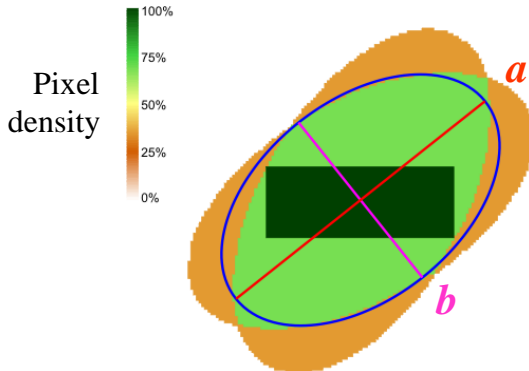
Rink, M. (1976). – A computerized quantitative image analysis procedure for investigating features and an adapted image process. *Journal of Microscopy*, 107: 267-286

Case of N objects

$$\mathbf{M} = \frac{1}{N} \sum_j \mathbf{M}_j = \frac{1}{N} \begin{vmatrix} \sum_j m_{xxj} & \sum_j m_{xyj} \\ \sum_j m_{xyj} & \sum_j m_{yyj} \end{vmatrix}$$

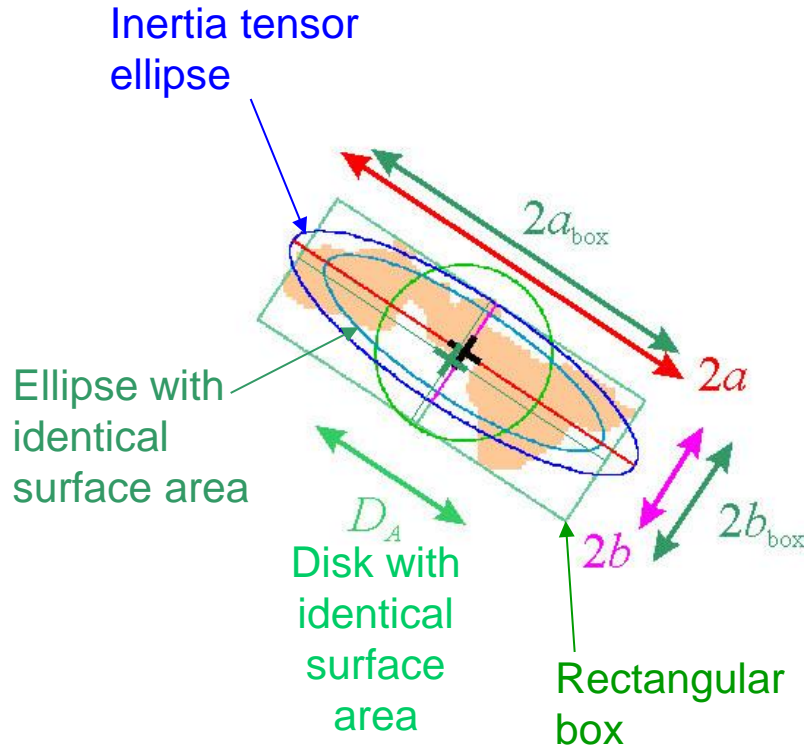
$$R = \frac{a}{b} = \sqrt{\frac{\rho_1}{\rho_2}}$$

P. Launeau
(2004) "Mise en évidence des écoulements magmatiques par analyse d'images 2-D des distributions 3-D d'Orientations Préférentielles de Formes". *Bull. Soc. Géol. Fr.*, 175, 331-350

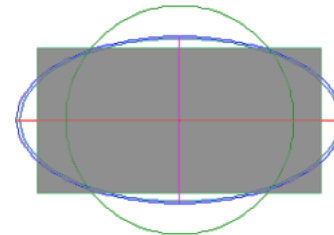


Stack of object piled on their gravity centre

Other shape parameters ...

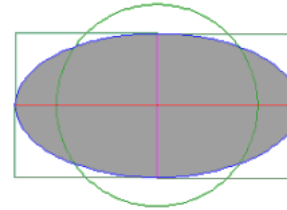


Shape n°



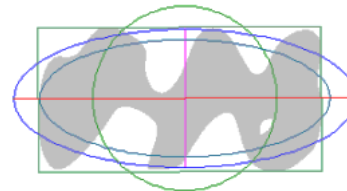
1 : rectangle

$r = 2$



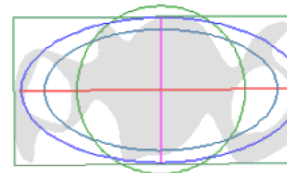
2 : ellipse

$r = 2$



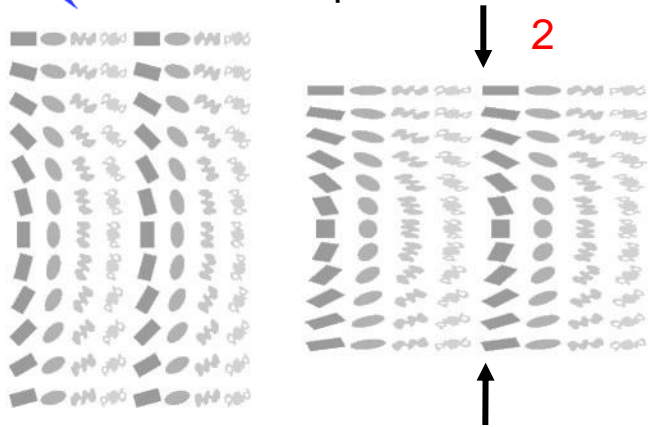
3 : "centrifugal" shape

$r = 2,4$



4 : "centripetal" shape

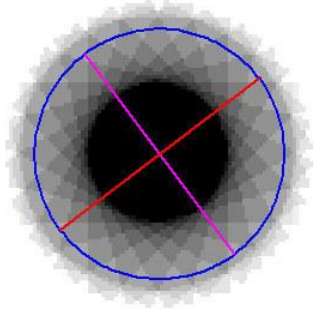
$r = 2$



Passive Deformation no viscosity contrast

Shape n° 1

A n=24 a=0,9278 cm b=0,9267 cm sr=1,001 [1,002]h [1,002]b , 52,83°
k=0,001, kn=0,002 {0,589}, kb=0,002 {0,583}

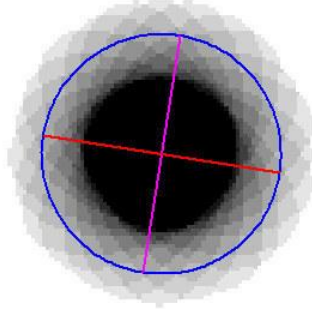


R

1

2

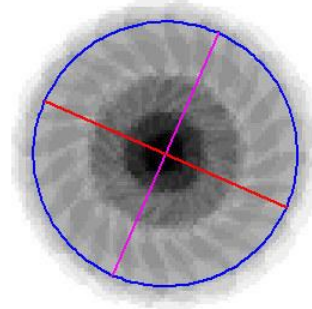
B n=24 a=0,8000 cm b=0,7994 cm sr=1,001 [1,001]h [1,001]b , 98,79°
k=0,001, kn=0,001 {0,590}, kb=0,001 {0,585}



1

3

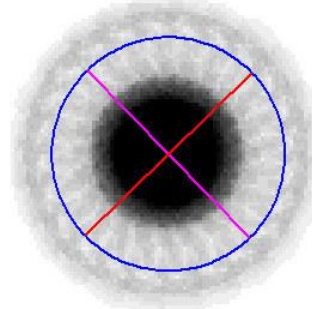
C n=24 a=0,9256 cm b=0,9252 cm sr=1,000 [1,001]h [1,001]b , 113,57°
k=0,000, kn=0,001 {0,714}, kb=0,001 {0,581}



1

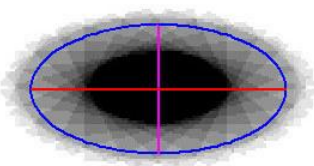
4

D n=24 a=0,8083 cm b=0,8079 cm sr=1,001 [1,001]h [1,001]b , 45,90°
k=0,001, kn=0,001 {0,589}, kb=0,001 {0,577}



1

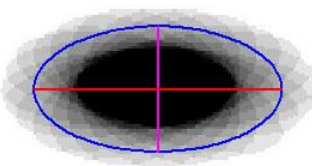
A n=24 a=1,3074 cm b=0,6567 cm sr=1,991 [2,999]h [3,047]b , 90,00°
k=0,597, kn=0,800 {0,746}, kb=0,806 {0,741}



R

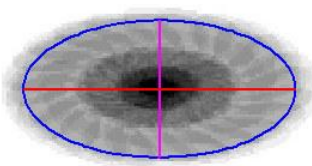
2

B n=24 a=1,1279 cm b=0,5697 cm sr=1,980 [2,970]h [3,034]b , 90,01°
k=0,593, kn=0,796 {0,745}, kb=0,804 {0,738}



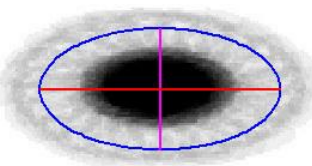
2

C n=24 a=1,3075 cm b=0,6563 cm sr=1,992 [2,530]h [3,088]b , 90,04°
k=0,598, kn=0,730 {0,819}, kb=0,810 {0,738}



2

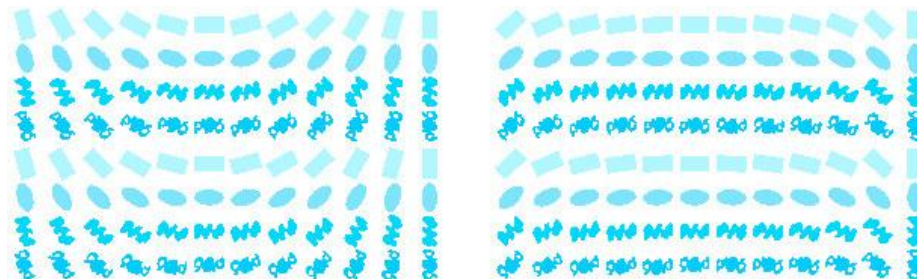
D n=24 a=1,1430 cm b=0,5756 cm sr=1,986 [2,971]h [3,023]b , 90,08°
k=0,595, kn=0,796 {0,748}, kb=0,803 {0,742}



2

Active Deformation

rigid body rotation



P.K. Harvey et C.C. Ferguson, 1978

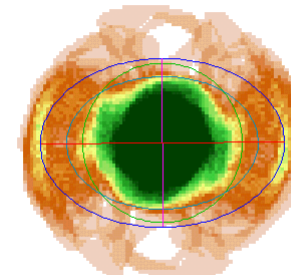
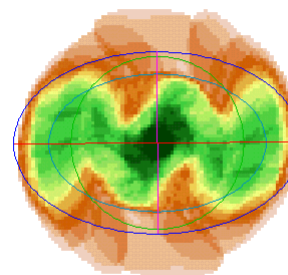
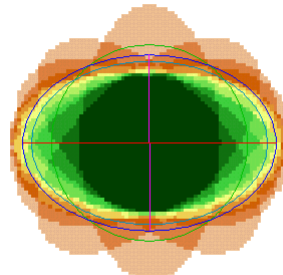
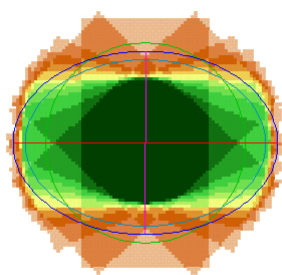
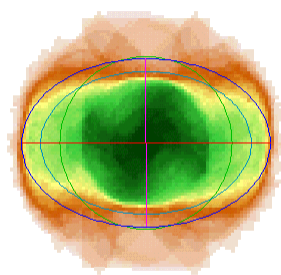
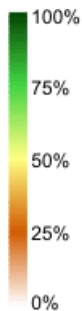
Shape n° 1234

1

2

3

4



R_{γ_c} 1,48

1,44

1,44

1,57

1,44

R_n 1,99

1,95

1,98

1,99

1,99

r 2

2

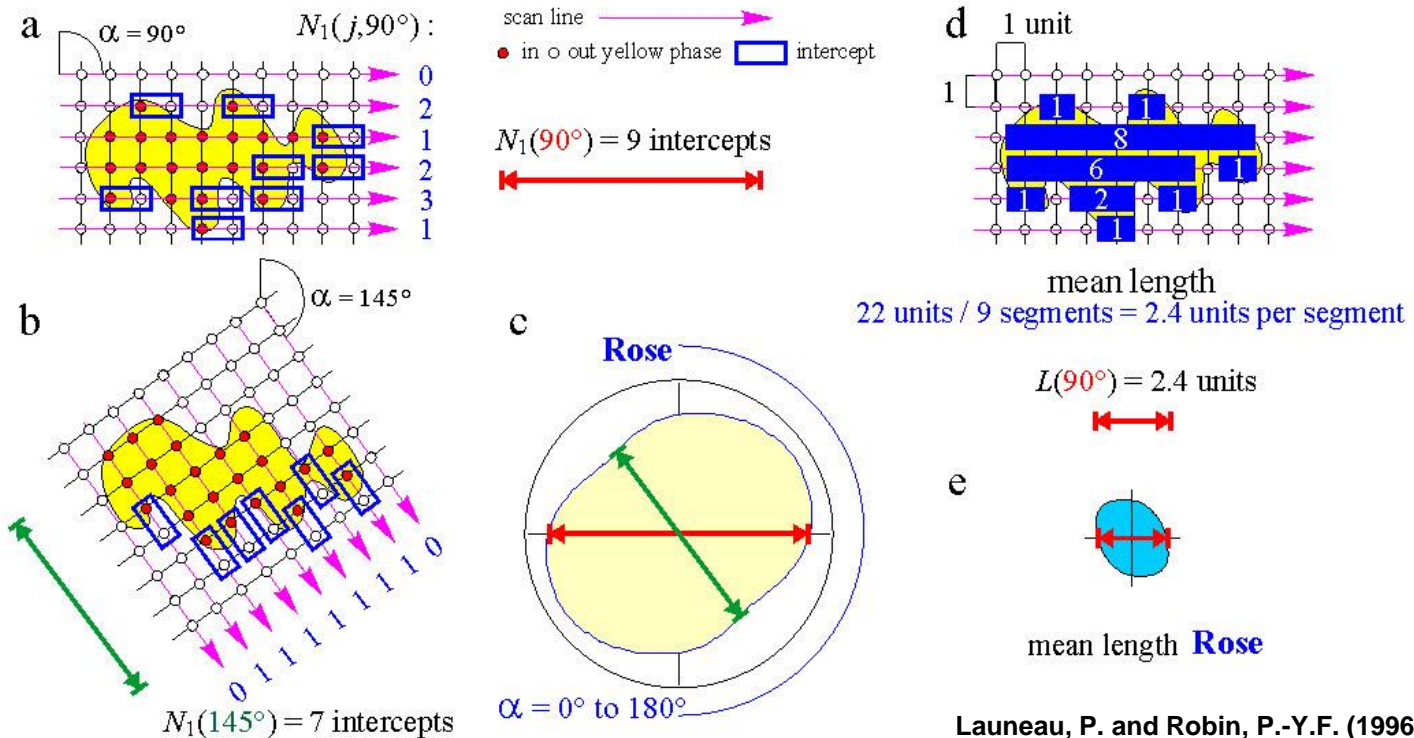
2

2,4

2

Case of 1 object

Shape analysis with the intercepts

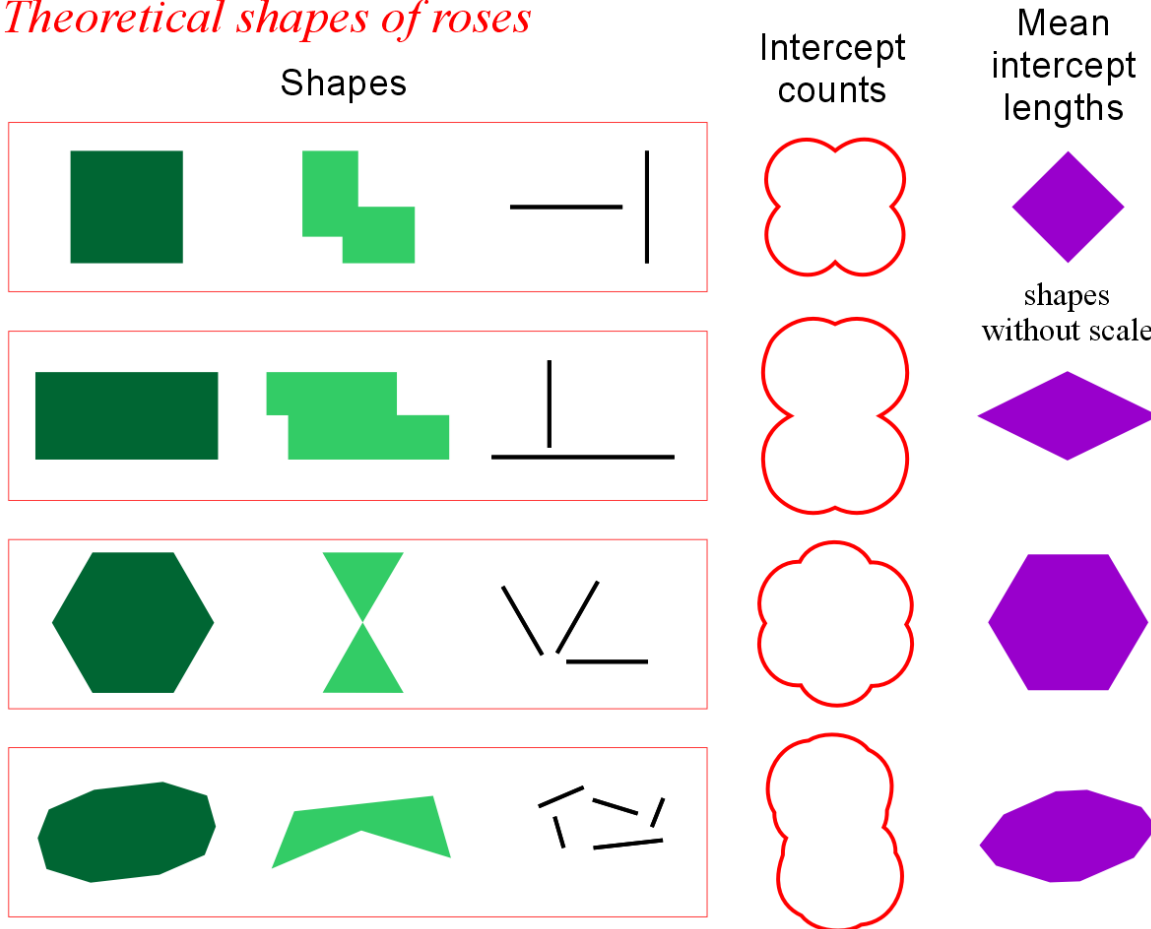


Launeau, P. and Robin, P.-Y.F. (1996). – Fabric analysis using the intercept method. *Tectonophysics* 267, 91-119

Case of 1 object

Shape analysis with the intercepts

Theoretical shapes of roses



shapes
without scale

Launeau, P. and Robin, P.-Y.F. (1996). – Fabric analysis using the intercept method. *Tectonophysics* 267, 91-119

Case of N objects

Shape analysis with the intercepts

A n=2 a=5,9926 cm b=3,9318 cm R=1,524 [1,729]n [1,734]b . 89,99°
K=0,398, Kn=0,498 {0,799}, Kb=0,501 {0,795}

A a=3,9357 cm b=2,5886 cm R=1,520 , 90,12°

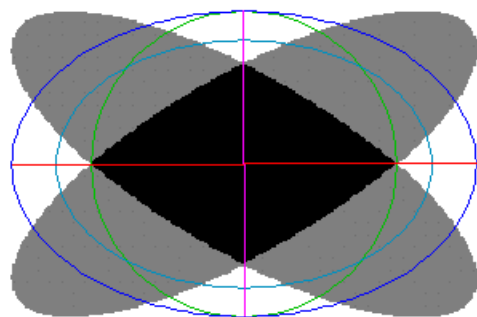
A a=11,9422 cm b=7,8547 cm R=1,520 , 90,12°

(1) 84°

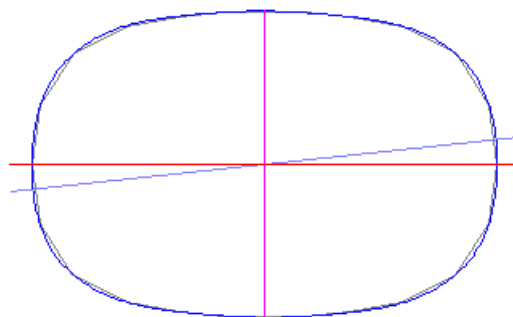
(1) 60°
(2) 120°

$r = 3$ $R = 1,52$

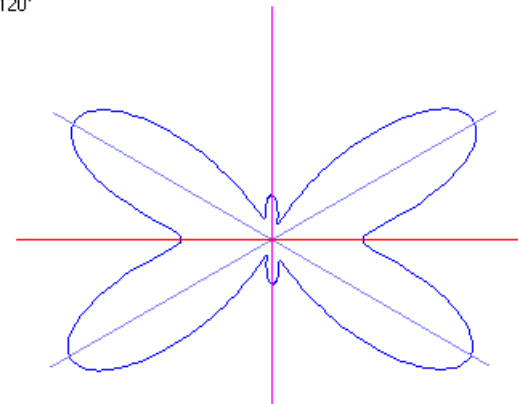
$R = 1,52$



Inertia tensor



Lengths of intercepts



Boundaries directions

A n=2 a=0,4883 cm b=0,3171 cm R=1,540 [1,732]n [1,746]b . 89,82°
K=0,407, Kn=0,500 {0,814}, Kb=0,506 {0,804}

A a=0,2599 cm b=0,1990 cm R=1,306 , 89,98°

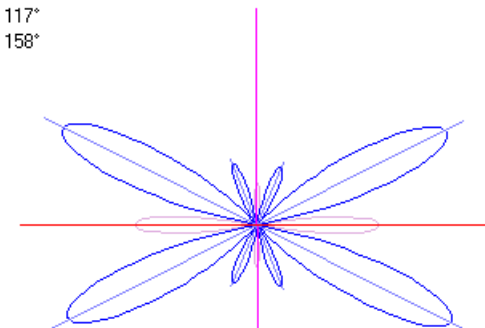
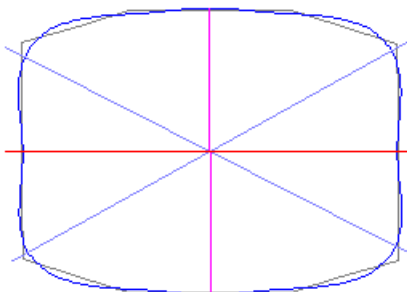
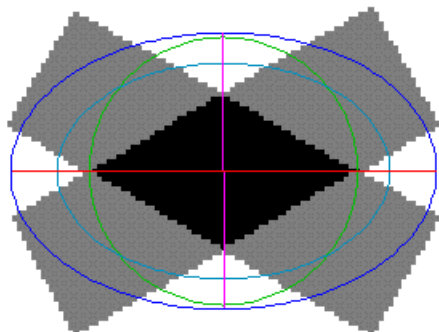
A a=0,9649 cm b=0,7434 cm R=1,298 , 89,98°

(1) 61°
(2) 117°

(1) 23°
(2) 63°
(3) 117°
(4) 158°

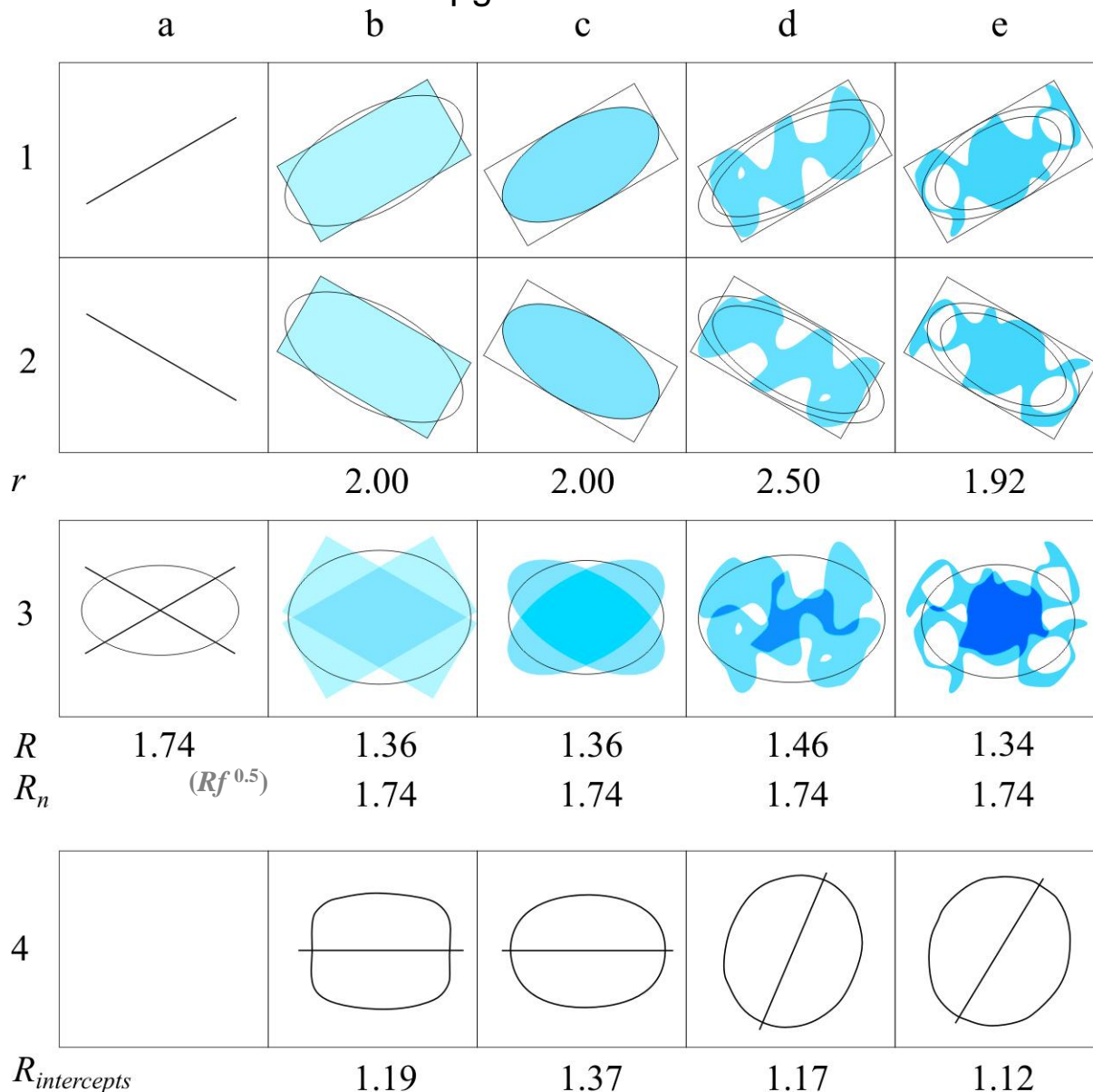
$r = 3$ $R = 1,54$

$R = 1,30$



Intercepts are sensitive to the boundary

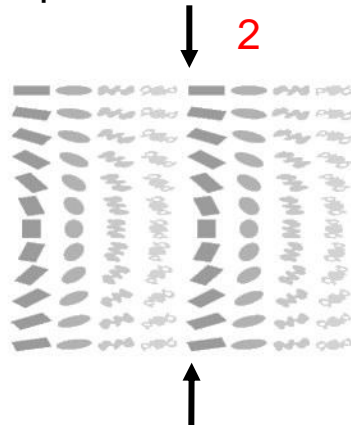
SPO of
objects
with any
boundary
geometry



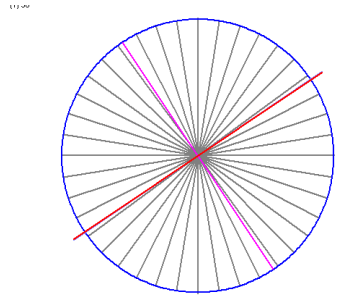
Inertia tensor 

intercepts 

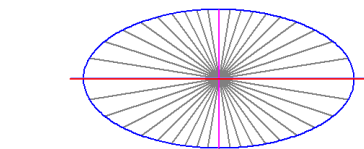
Passive Deformation no viscosity contrast



Shape n° 1



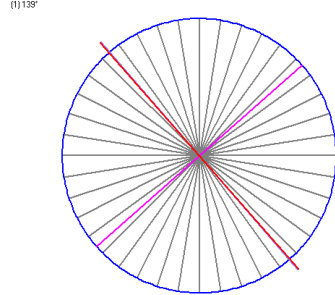
R $a=0.6328$ cm $b=0.3235$ cm $R=1.959$, 90.05°
(1) 90°



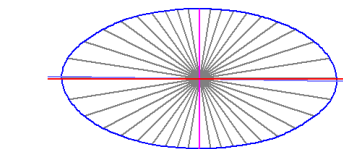
R 1.96

0.1 cm

Shape n° 2



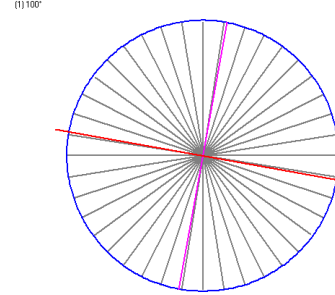
B $a=0.4191$ cm $b=0.4188$ cm $R=1.001$, 136.64°
(1) 136°



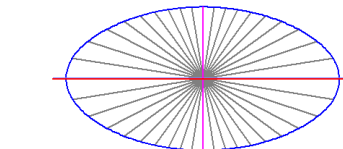
B 1.97

0.1 cm

Shape n° 3



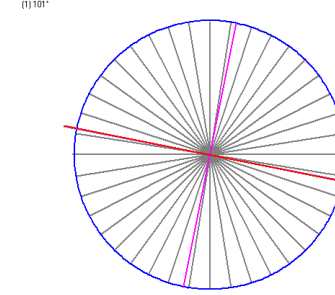
C $a=0.2336$ cm $b=0.2317$ cm $R=1.008$, 100.27°
(1) 100°



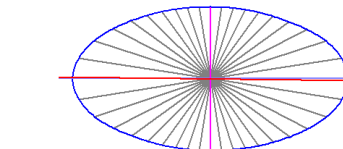
C 1.91

0.1 cm

Shape n° 4



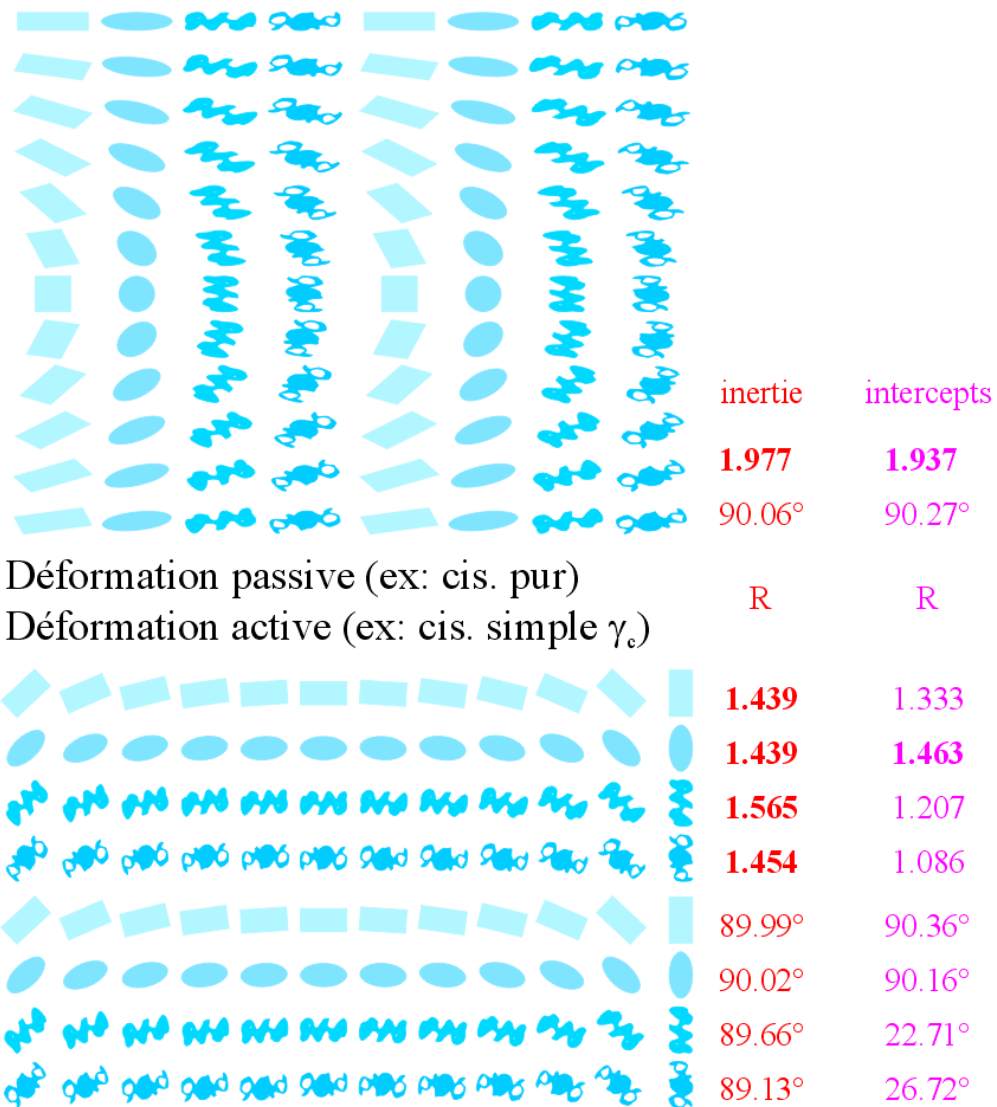
D $a=0.1570$ cm $b=0.1558$ cm $R=1.008$, 101.31°
(1) 101°



D 1.92

0.1 cm

Passive / Active Deformation

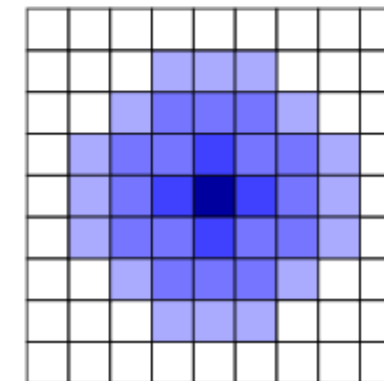
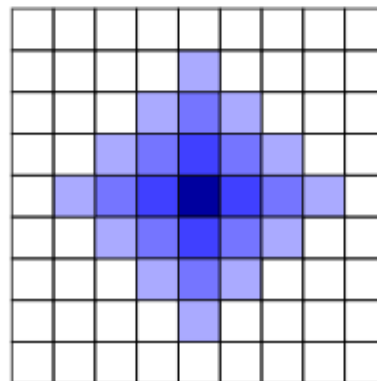
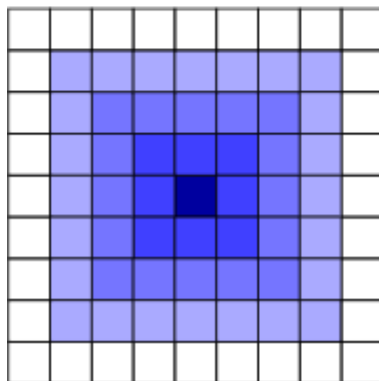
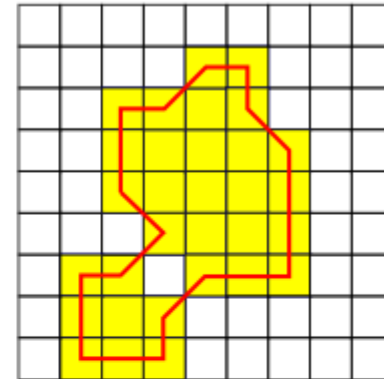
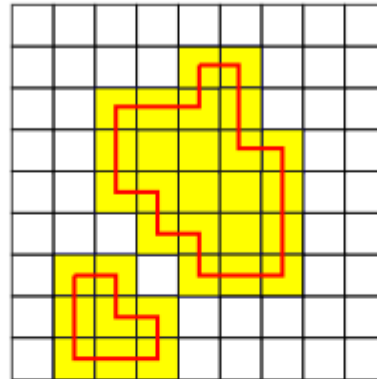
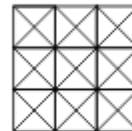
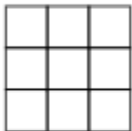
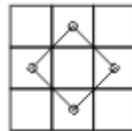
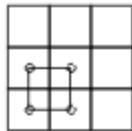


Application to digital images

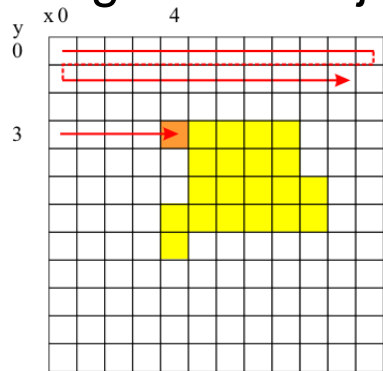
Connexity

4

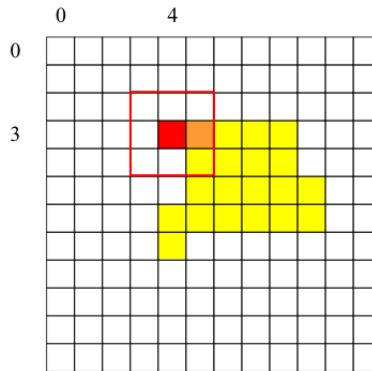
8



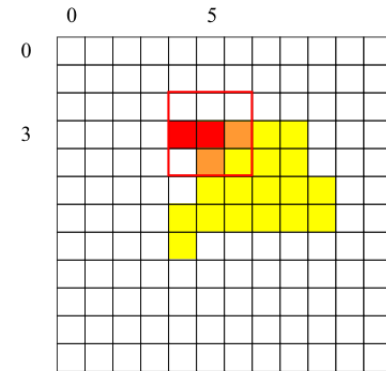
Scanning of one object



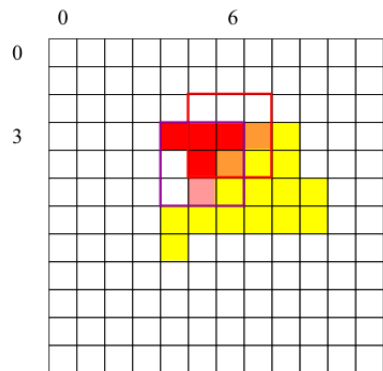
x, y x, y
4, 3



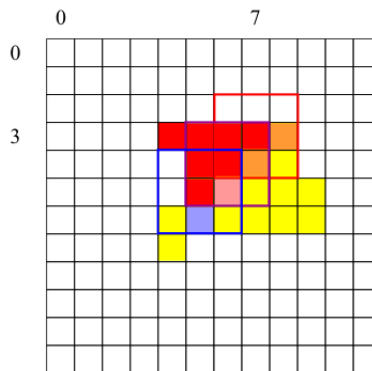
x, y x, y
4, 3 5, 3



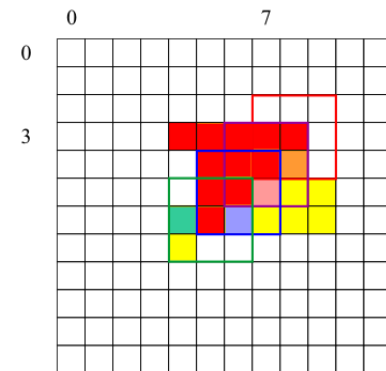
x, y x, y
5, 3 6, 3
5, 4



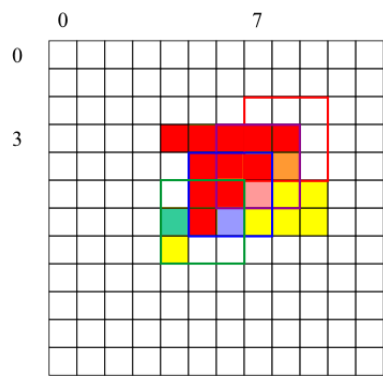
x, y x, y
6, 3 — 7, 3
5, 4 — 6, 4
5, 5



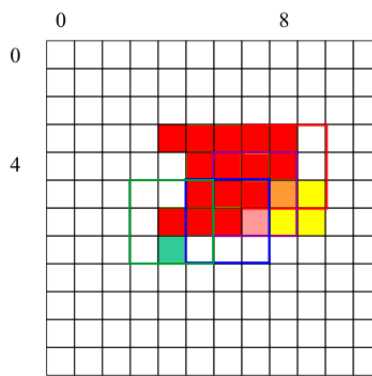
x, y x, y
7, 3 — 8, 3
6, 4 — 7, 4
5, 5 — 6, 5
5, 6



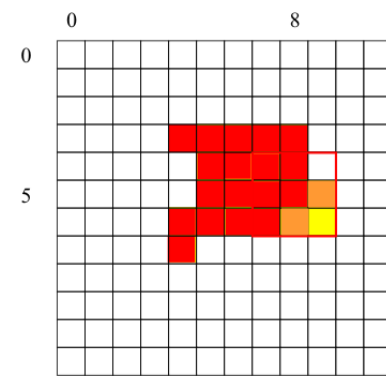
x, y x, y
8, 3 — 8, 4
7, 4 — 7, 5
6, 5 — 6, 6
5, 6 — 4, 6



x, y x, y
8, 3 — 8, 4
7, 4 — 7, 5
6, 5 — 6, 6
5, 6 — 4, 6

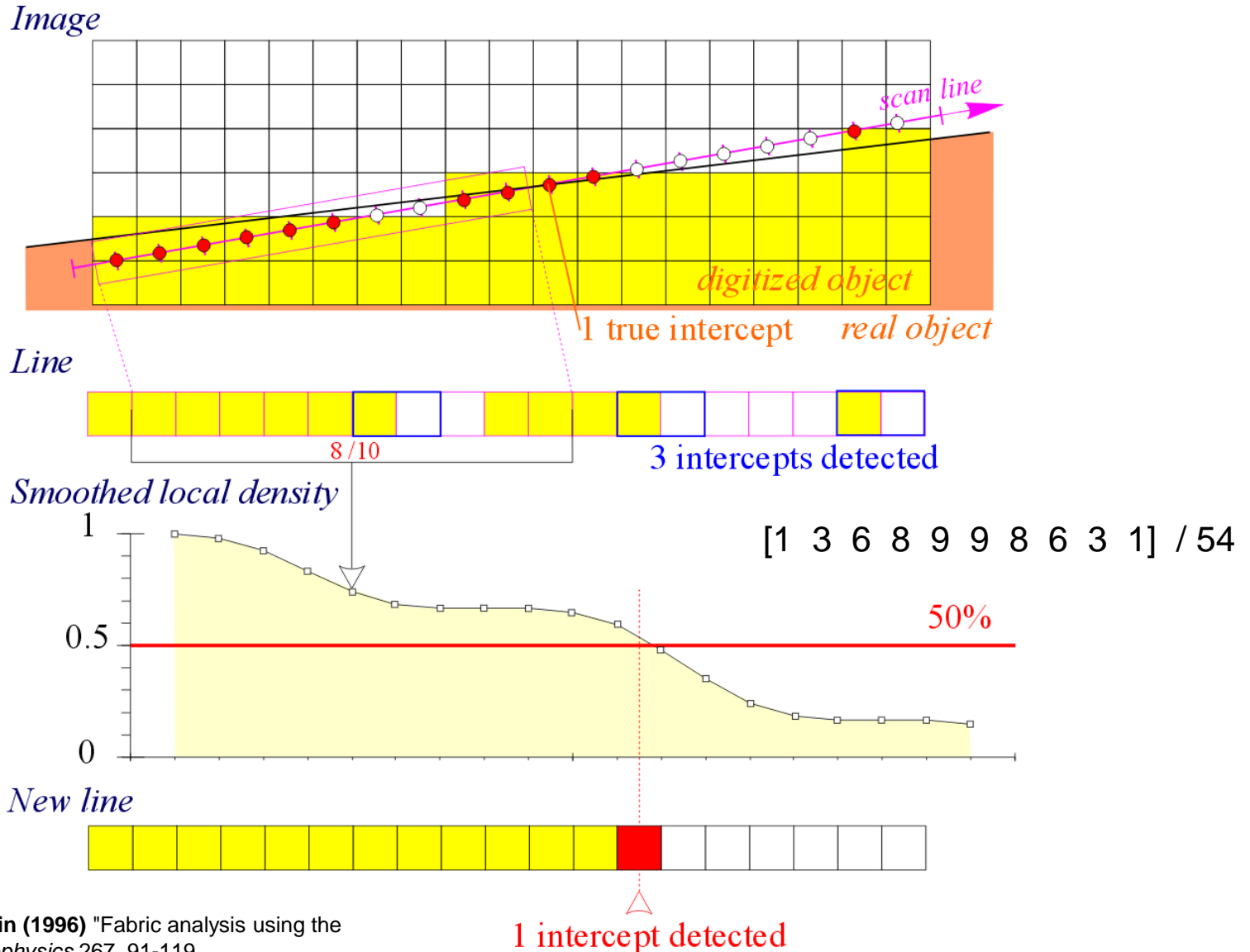


x, y x, y
8, 4 — 8, 5
7, 5 — 7, 6
6, 6 — 4, 7
4, 6



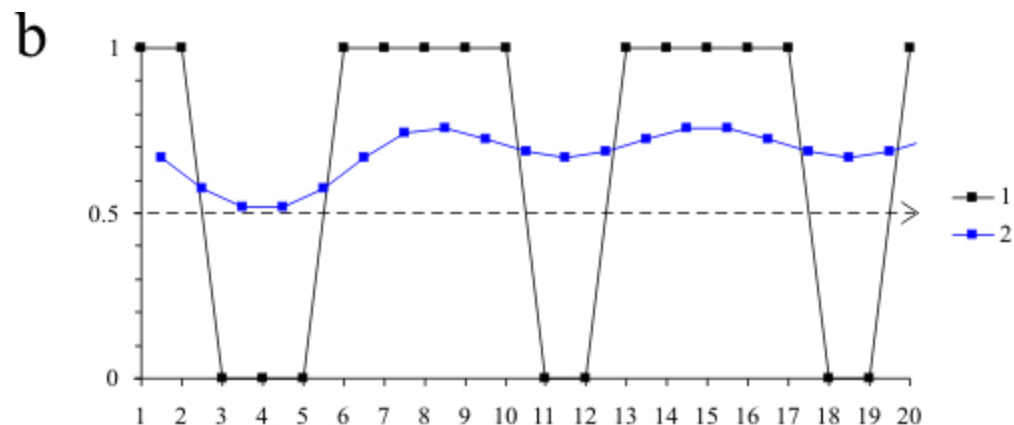
x, y x, y
8, 5 — 9, 5
7, 6 — 8, 6
4, 7

Density of intercept count

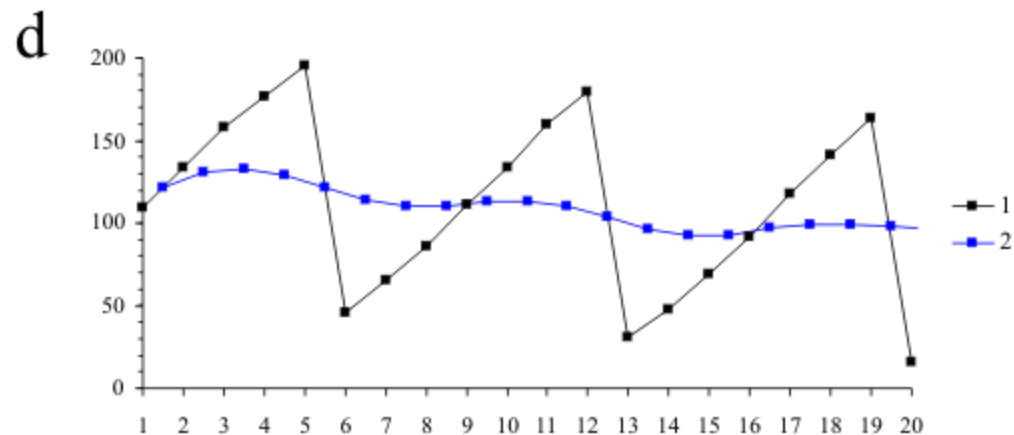


- 1 – resampling along any line direction in grey level
- 2 – smoothing sampled line with $[1\ 3\ 6\ 8\ 9\ 9\ 8\ 6\ 3\ 1] / 54$ minimized over counts of intercepts

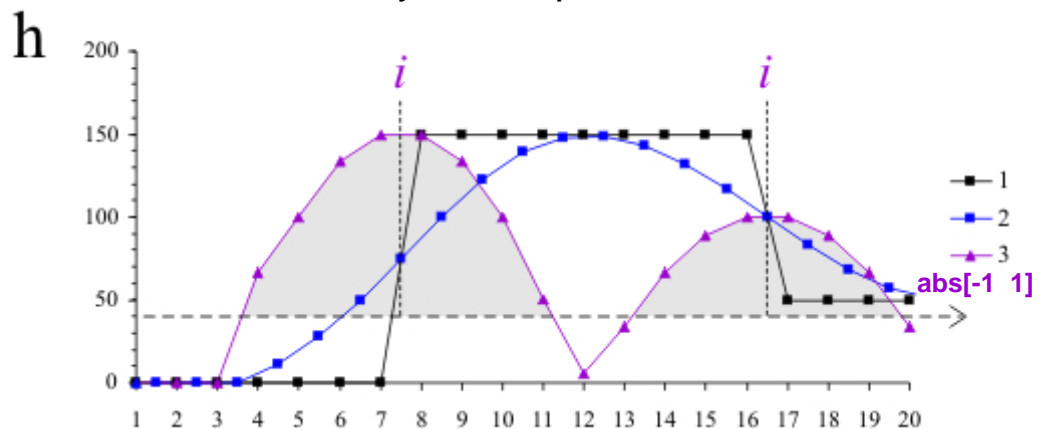
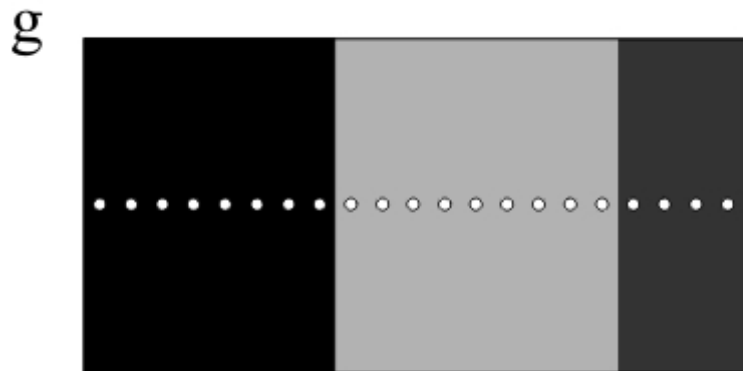
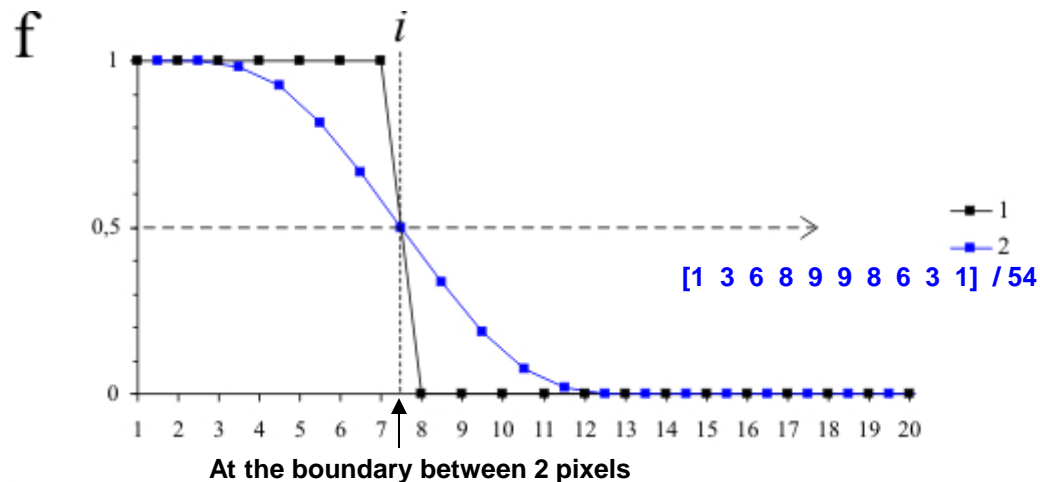
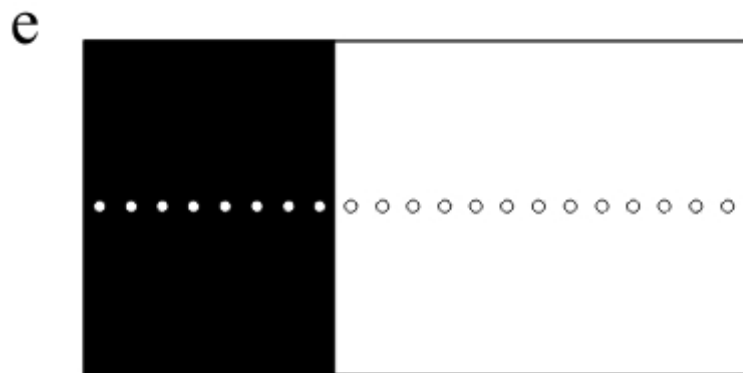
Thresholded (binary) image



Grey level image

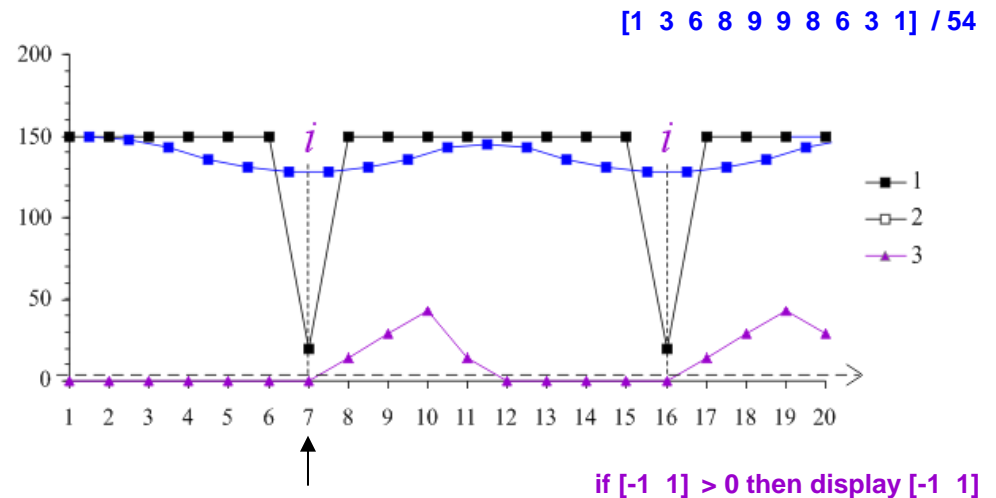
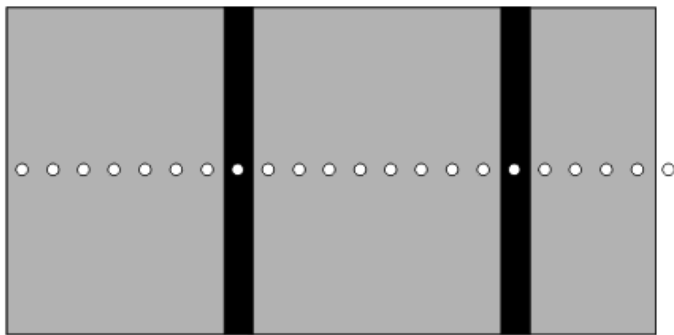


Intercept detection i in grey levels



Intercept detection i in grey levels (sp. case)

In case of one grey phase with dark boundaries the intercepts detection occurs only from a dark pixel to a bright pixel to avoid double detection on both sides of each boundary



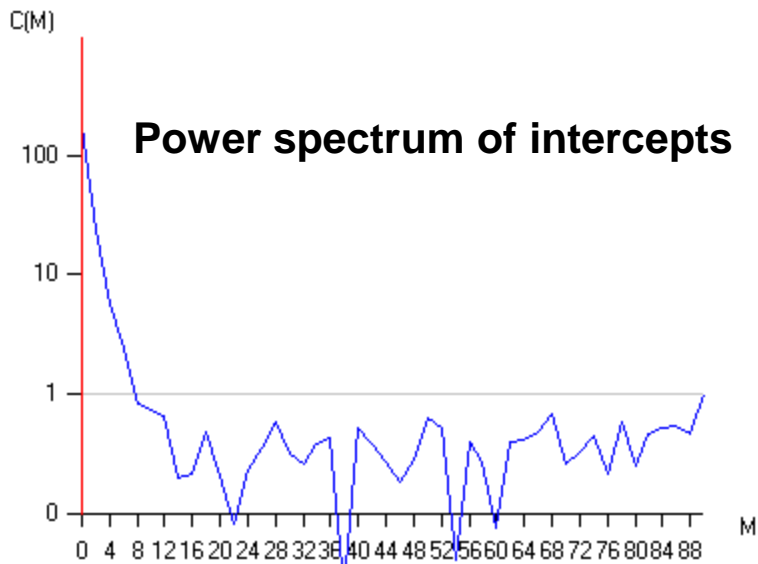
Fourier series of intercepts

Hilliard, J. E., 1962. Specification and measurement of microstructural anisotropy. Trans. of the Metallurgical Society of AIME, 224: 1201-1211.

$$N_1(\alpha) = \sum_j N_1(j, \alpha)$$

Fourier series of intercepts counts :

$$A_{2m} = \frac{2}{K} \sum_{k=0}^{K-1} N_L(k\delta\alpha) \cos 2mk\delta\alpha, \quad B_{2m} = \frac{2}{K} \sum_{k=0}^{K-1} N_L(k\delta\alpha) \sin 2mk\delta\alpha$$



phase power

$$\tan 2m\varphi_{2m} = \frac{B_{2m}}{A_{2m}}, \quad C_{2m}^2 = A_{2m}^2 + B_{2m}^2$$

0.1 cm

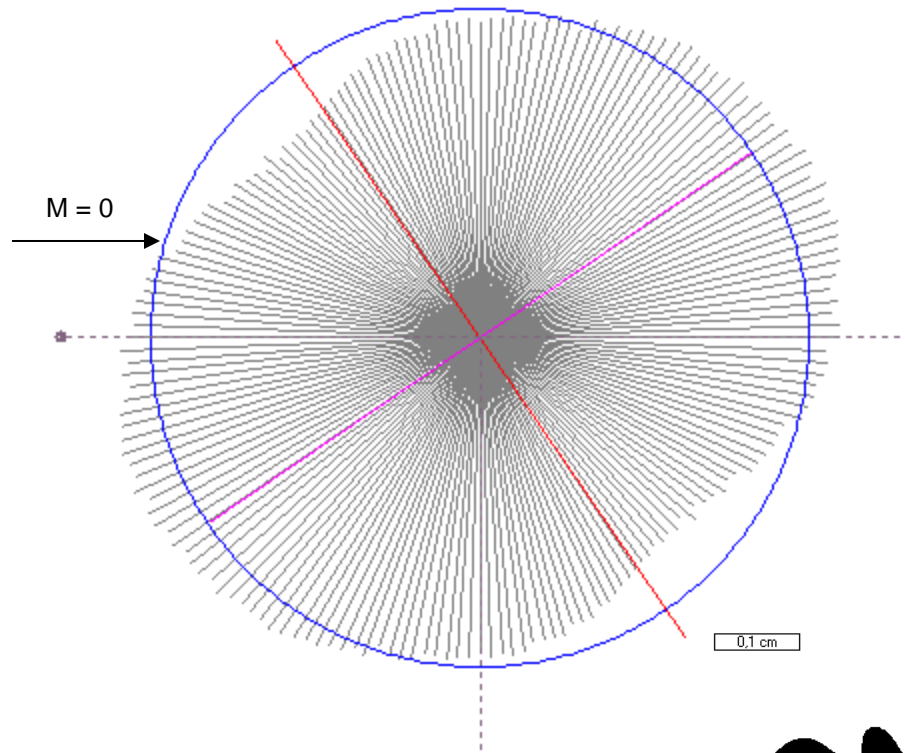
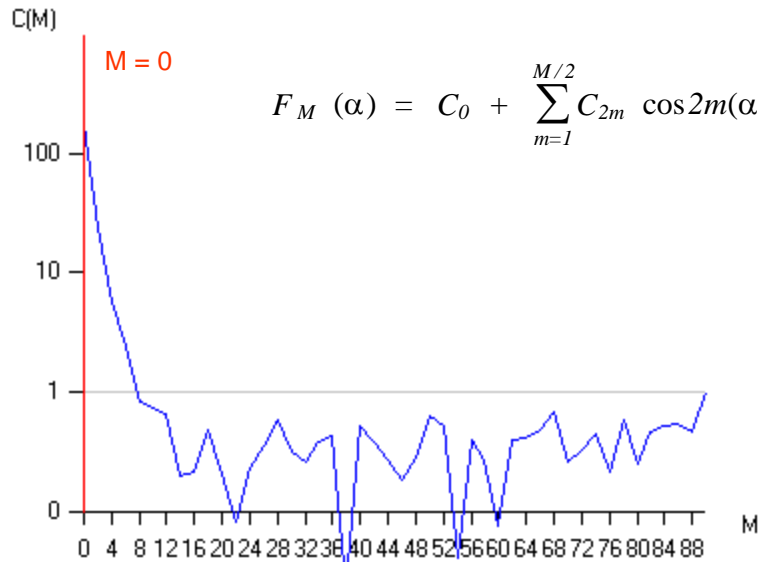


Fourier series : rose of intercepts

A a=0,2592 cm b=0,2592 cm R=1,000 , 145,62° , angle X: 124,38°

$$N_1(\alpha) = \sum_j N_1(j, \alpha)$$

$$F_M(\alpha) = C_0 + \sum_{m=1}^{M/2} C_{2m} \cos 2m(\alpha - \varphi_{2m})$$



$$F_M(\alpha) = C_0 + \sum_{m=1}^{M/2} (A_{2m} \cos 2m\alpha + B_{2m} \sin 2m\alpha)$$

0,1 cm



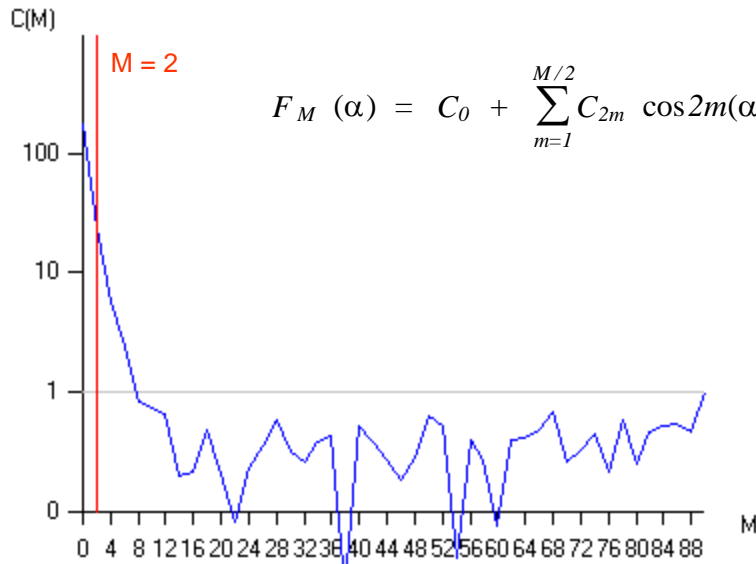
Fourier series : rose of intercepts

A a=0,2942 cm b=0,2242 cm R=1,312 , 145,62° , angle X: 124,38°

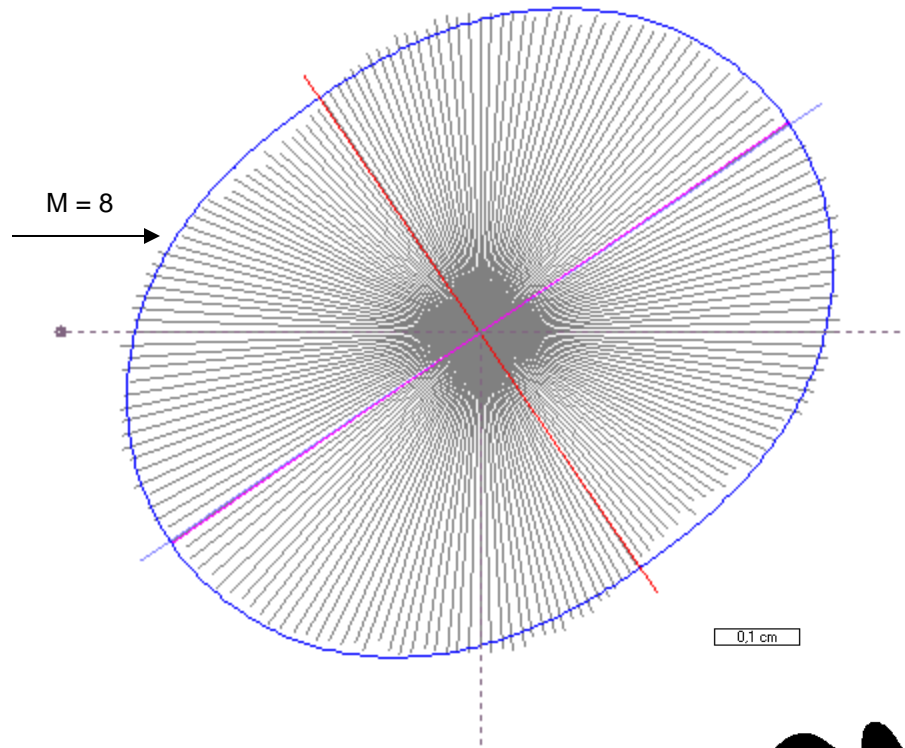
(1) 56°

$$N_1(\alpha) = \sum_j N_1(j, \alpha)$$

$$F_M(\alpha) = C_0 + \sum_{m=1}^{M/2} C_{2m} \cos 2m(\alpha - \varphi_{2m})$$



M = 8



$$F_M(\alpha) = C_0 + \sum_{m=1}^{M/2} (A_{2m} \cos 2m\alpha + B_{2m} \sin 2m\alpha)$$

0.1 cm



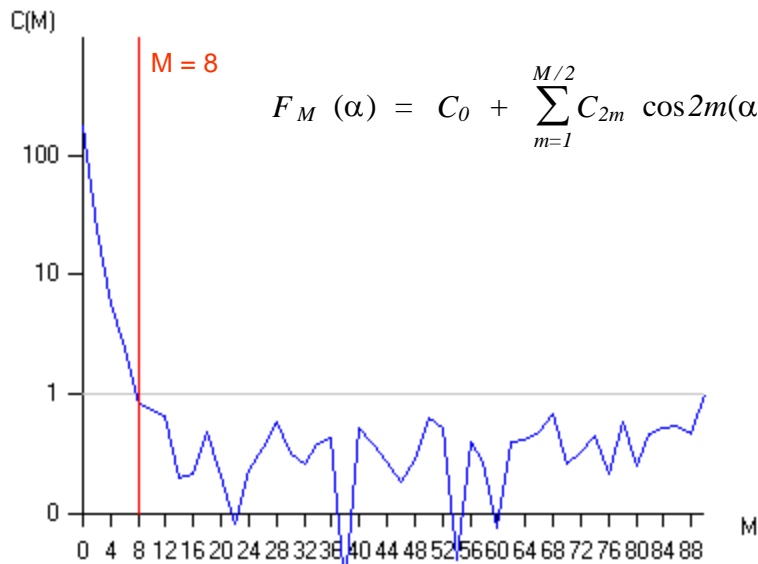
Fourier series : rose of intercepts

A a=0,2924 cm b=0,2212 cm R=1,322 , 145,62° , angle X: 124,38°

(1) 65°

$$N_1(\alpha) = \sum_j N_1(j, \alpha)$$

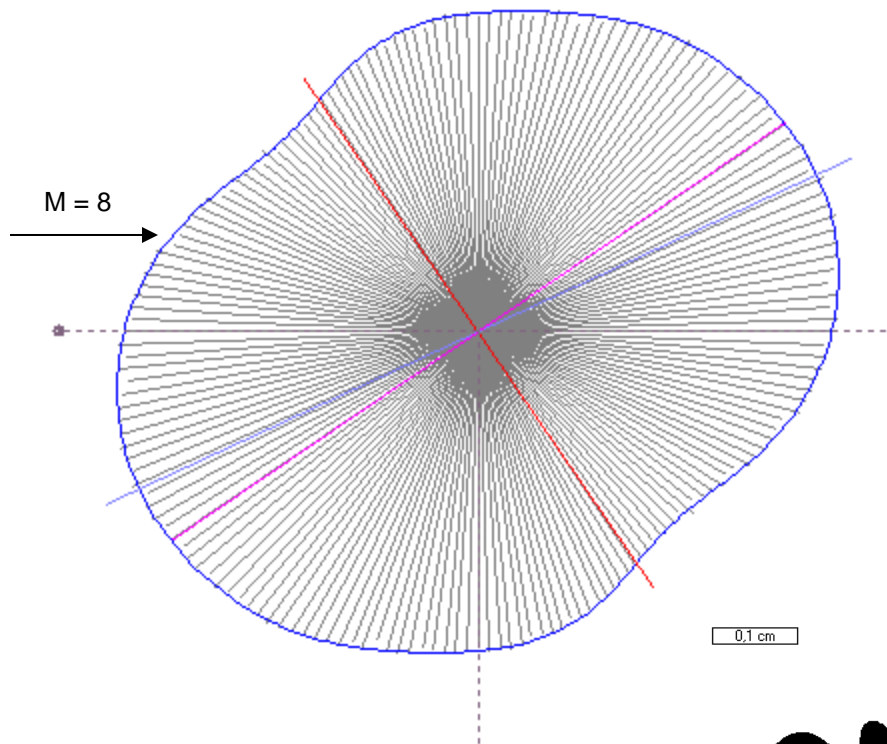
$$F_M(\alpha) = C_0 + \sum_{m=1}^{M/2} C_{2m} \cos 2m(\alpha - \varphi_{2m})$$



M = 8

$$F_M(\alpha) = C_0 + \sum_{m=1}^{M/2} (A_{2m} \cos 2m\alpha + B_{2m} \sin 2m\alpha)$$

0,1 cm



Fourier series : rose of traverses (mean length)

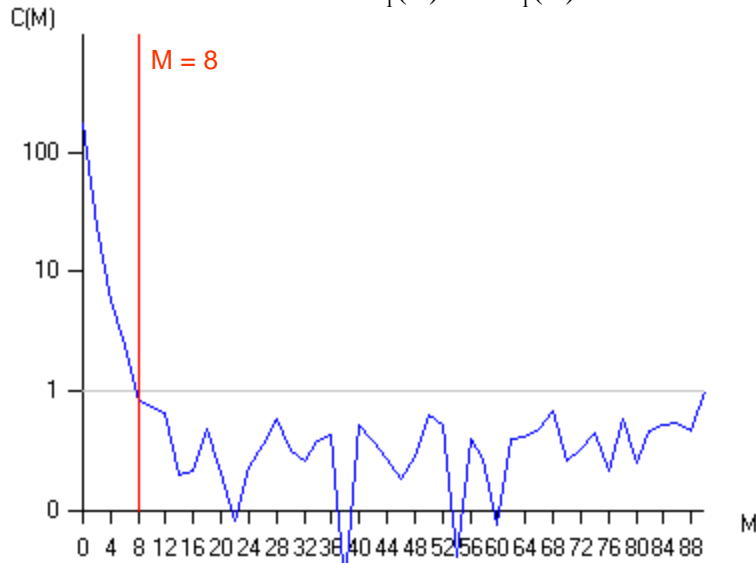
A a=0,1171 cm b=0,0886 cm R=1,322 , 145,62° , angle X: 124,38°

0.1 cm

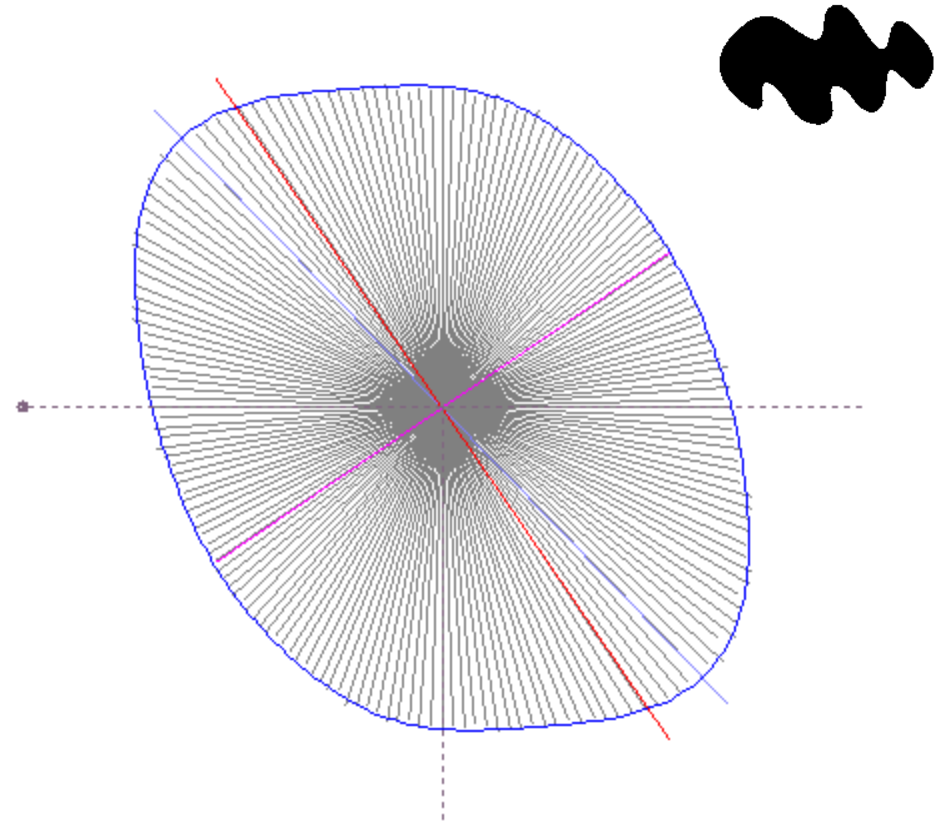
(1) 136°

$$N_1(\alpha) = \sum_j N_1(j, \alpha)$$

$$\bar{L}(\alpha) = \frac{A}{J N_1(\alpha)} = I \frac{N_0}{N_1(\alpha)}$$



$$F_M(\alpha) = C_0 + \sum_{m=1}^{M/2} (A_{2m} \cos 2m\alpha + B_{2m} \sin 2m\alpha)$$



Fourier series : rose of directions

For one rod :

$$D(\alpha) = L / |\sin(\alpha - \psi)| = \frac{L}{\sqrt{2}} \sqrt{1 - \cos 2(\alpha - \psi)}$$

$$D(\alpha) + D''(\alpha) = 0 \quad \text{for } \alpha \neq \psi \text{ modulo } \pi$$

$$\infty \quad \text{for } \alpha = \psi \text{ modulo } \pi$$

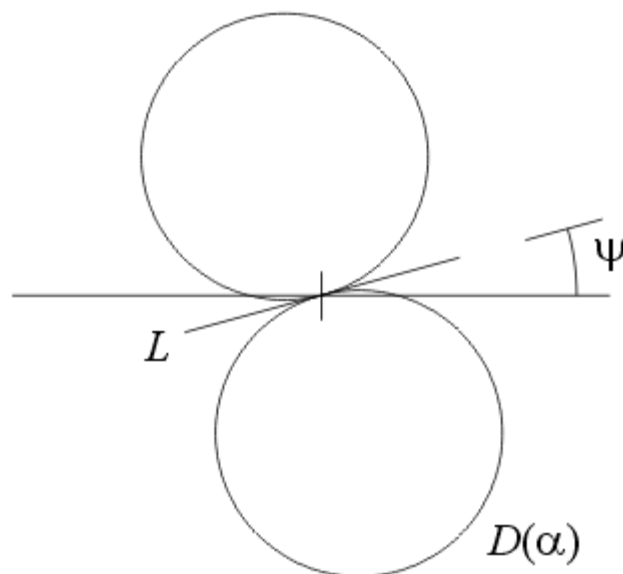
Since the first derivative $D'(\alpha)$ changes from $-L$ to $+L$ on either side of the singularity, $D(\alpha) + D''(\alpha)$ for one rod is a Dirac 'function', $2L \delta(\alpha - \psi)$, i.e. with an integrated value of $2L$:

$$D(\alpha) + D''(\alpha) = 2L \delta(\alpha - \psi)$$

For a population of rods :

$$N_L(\alpha) = \frac{D(\alpha)}{A^w} = \frac{1}{A^w} \sum_{i=1}^P L^i / |\sin(\alpha - \psi^i)|$$

$$N_L(\alpha) + N_L''(\alpha) = \frac{2}{A^w} \sum_{i=1}^P L^i \delta(\alpha - \psi^i)$$



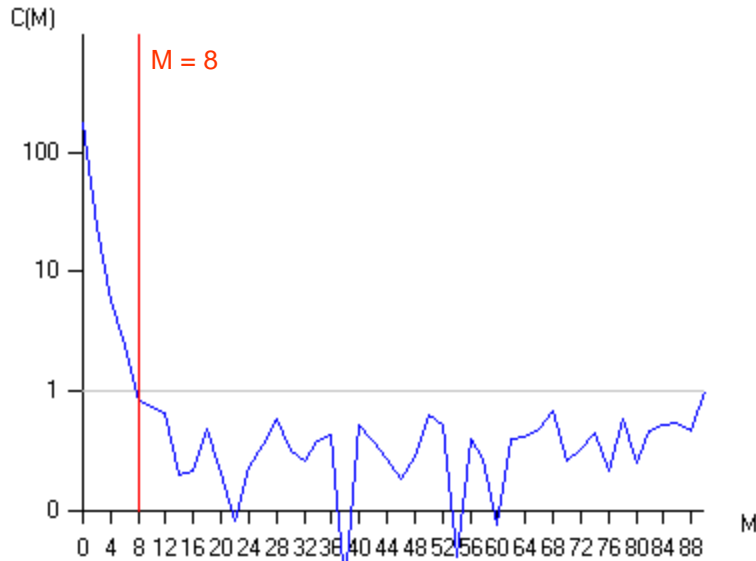
Hilliard, J. E., 1962. Specification and measurement of microstructural anisotropy. *Trans. of the Metallurgical Society of AIME*, 224: 1201-1211.

Fourier series : rose of directions

For a population of rods :

$$N_L(\alpha) = \frac{D(\alpha)}{A^w} = \frac{1}{A^w} \sum_{i=1}^P L^i / \sin(\alpha - \psi^i)$$

$$N_L(\alpha) + N_L''(\alpha) = \frac{2}{A^w} \sum_{i=1}^P L^i \delta(\alpha - \psi^i)$$



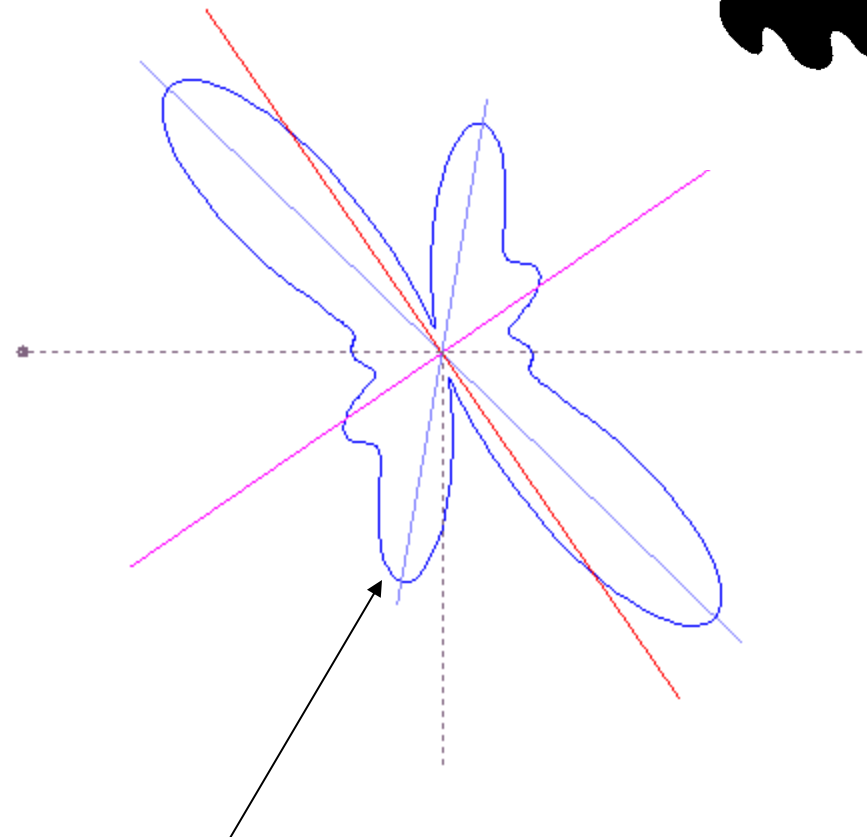
$$N_L''(\alpha) \approx F_M''(\alpha) = - \sum_{m=1}^{M/2} (2m)^2 (A_{2m} \cos 2m\alpha + B_{2m} \sin 2m\alpha)$$

$$\frac{2I(\alpha)}{A^w} \approx F_M(\alpha) + F_M''(\alpha) = C_0 + \sum_{m=1}^{M/2} [1 - (2m)^2] (A_{2m} \cos 2m\alpha + B_{2m} \sin 2m\alpha)$$

A Nla=2,069 cm⁻¹ Nlb=2,735 cm⁻¹ R=1,322 , 145,62° , angle X: 124,38°

0.1 cm

(1) 134°
(2) 10°



Fourier series : characteristic shape

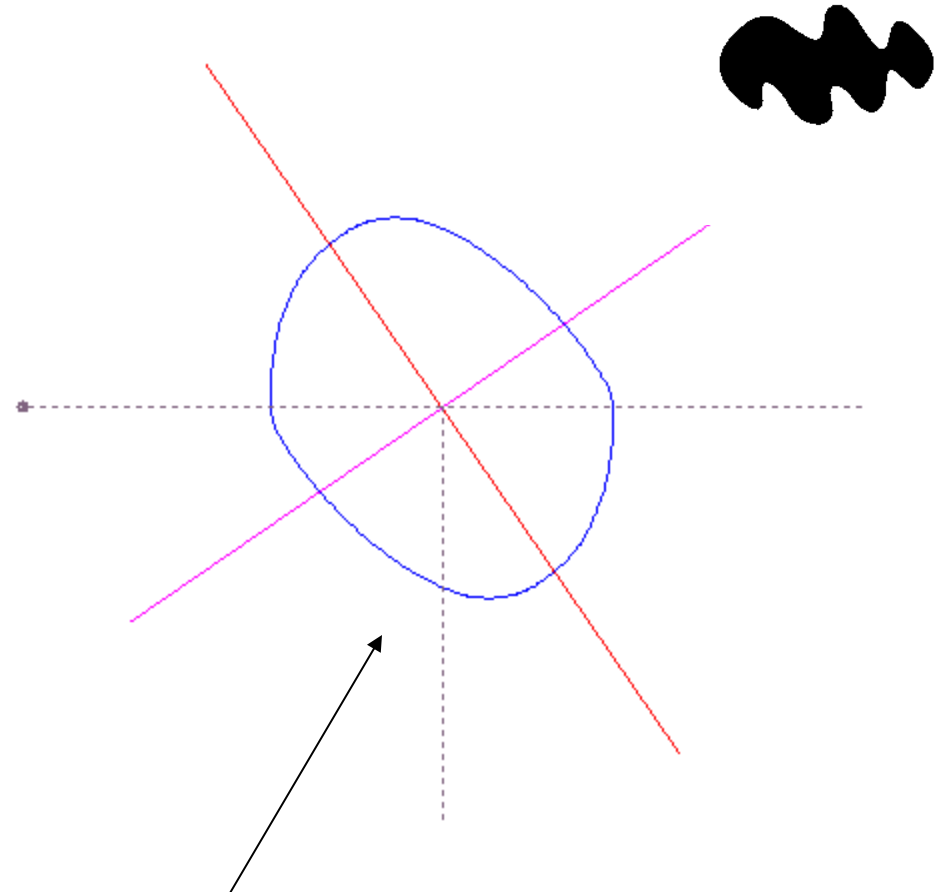
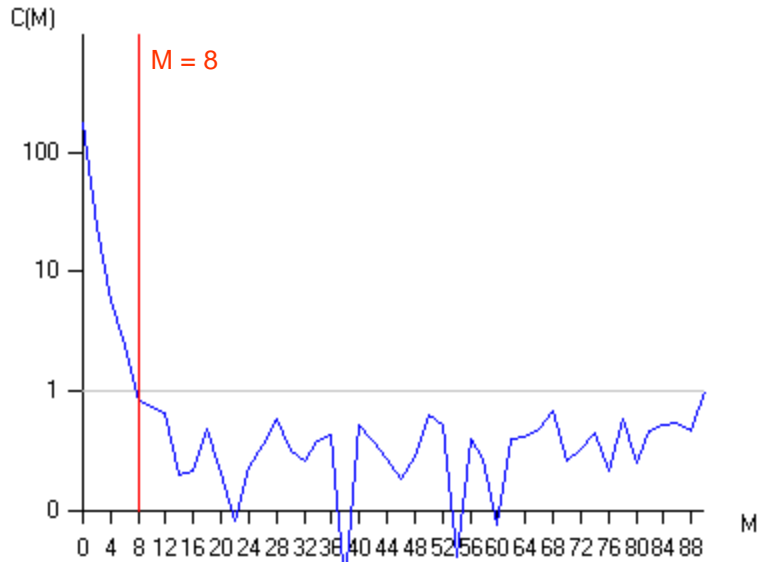
A R=1,322 , 145,62° , angle X: 124,38°

0.1 cm

For a population of rods :

$$N_L(\alpha) = \frac{D(\alpha)}{A^w} = \frac{1}{A^w} \sum_{i=1}^P L^i / \sin(\alpha - \psi^i)$$

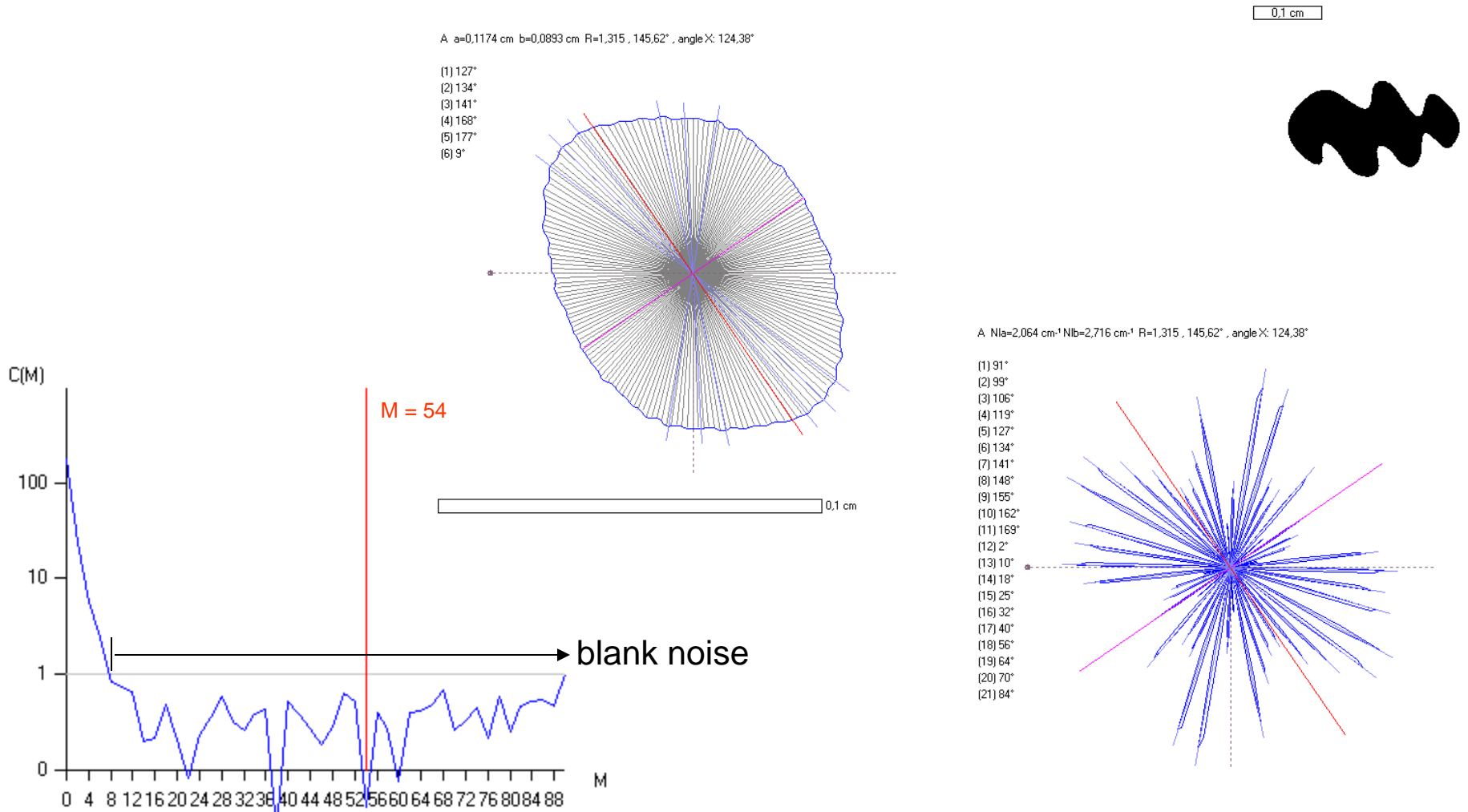
$$N_L(\alpha) + N_L''(\alpha) = \frac{2}{A^w} \sum_{i=1}^P L^i \delta(\alpha - \psi^i)$$



$$N_L''(\alpha) \approx F_M''(\alpha) = - \sum_{m=1}^{M/2} (2m)^2 (A_{2m} \cos 2m\alpha + B_{2m} \sin 2m\alpha)$$

$$\frac{2I(\alpha)}{A^w} \approx F_M(\alpha) + F_M''(\alpha) = C_0 + \sum_{m=1}^{M/2} [1 - (2m)^2] (A_{2m} \cos 2m\alpha + B_{2m} \sin 2m\alpha)$$

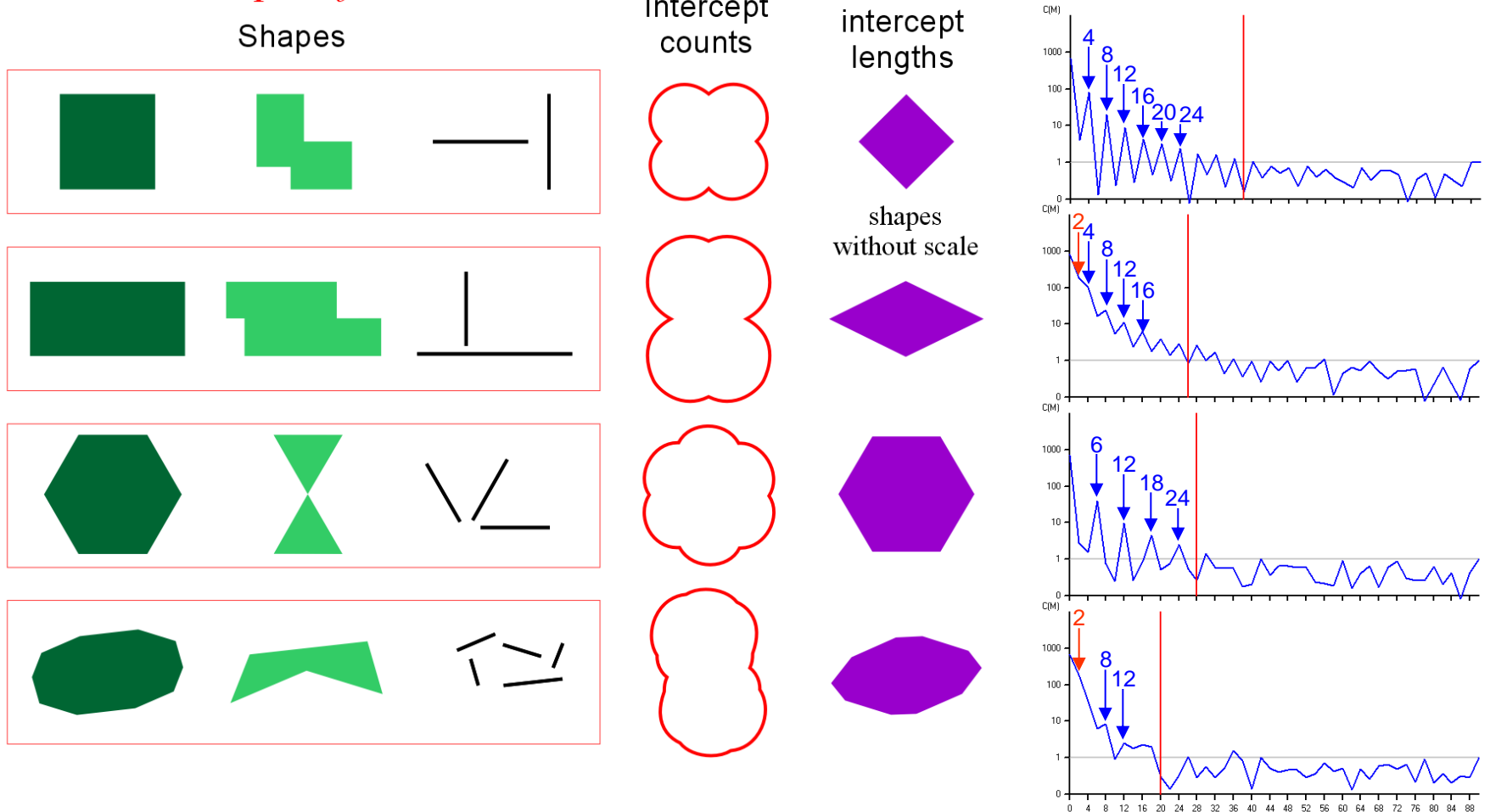
Fourier series : noise



Case of 1 object

Symmetry analysis with the Fourier power spectrum

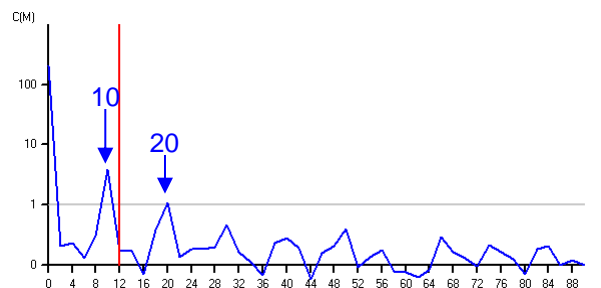
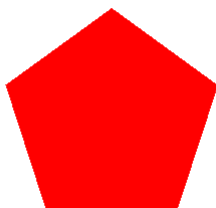
Theoretical shapes of roses



Case of 1 object

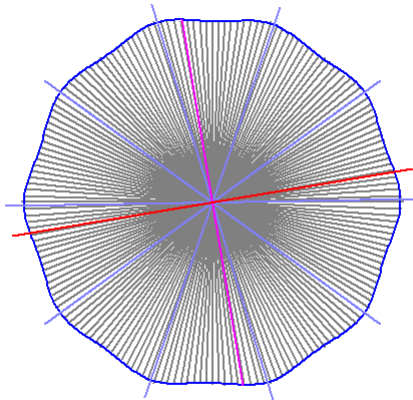
Symmetry analysis with the Fourier power spectrum

C_2 = anisotropy
 C_M = symmetry



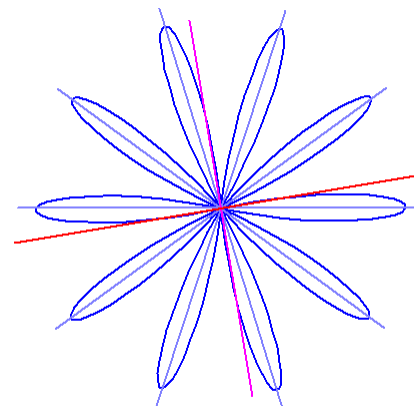
A a=0,5060 cm b=0,5072 cm R=0,998 , 80,42°

- (1) 126°
- (2) 163°
- (3) 19°
- (4) 54°
- (5) 89°



A N1a=0,138 cm² N1b=0,137 cm² R=0,998 , 80,42°

- (1) 90°
- (2) 126°
- (3) 163°
- (4) 18°
- (5) 54°

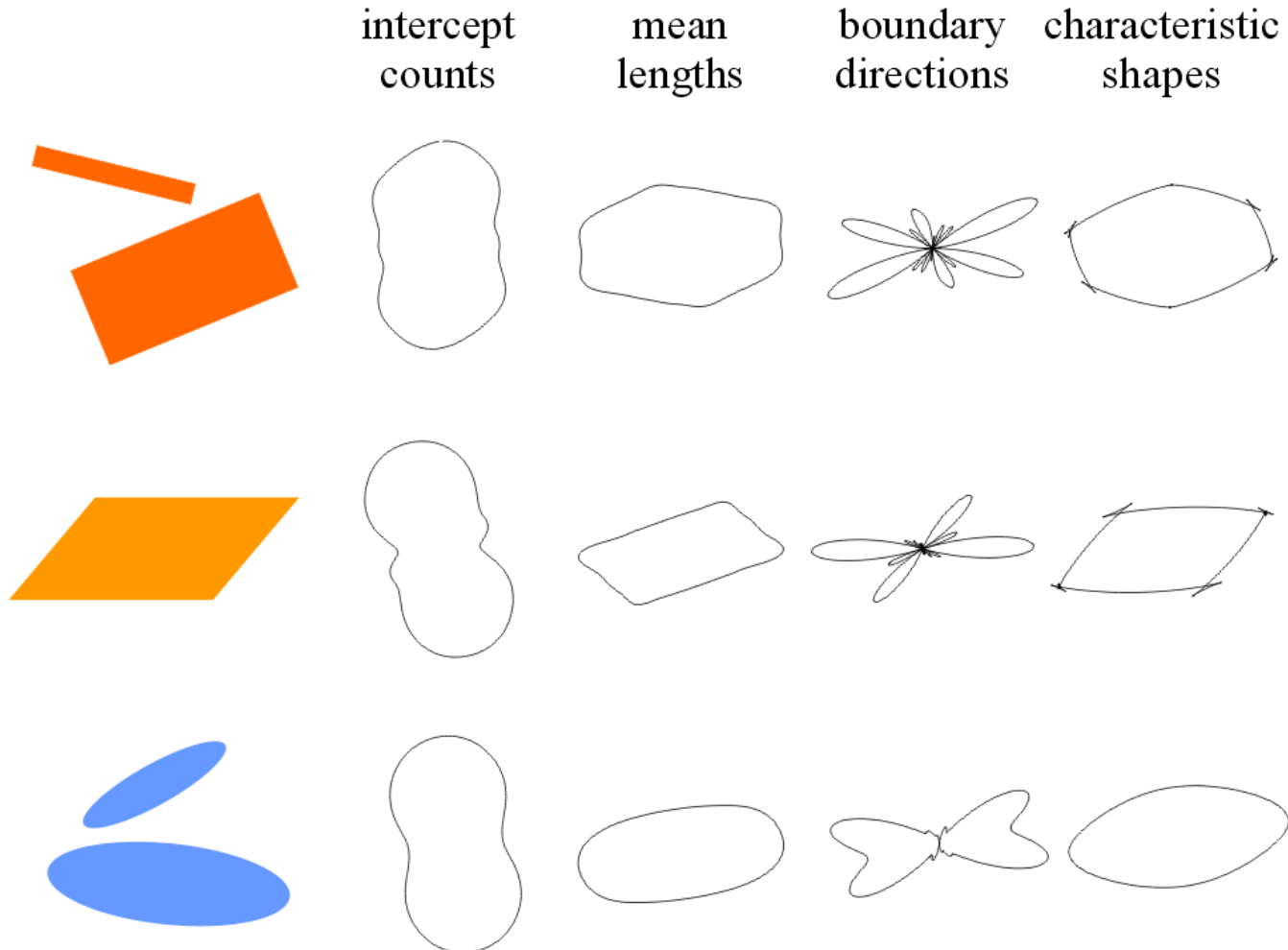


0,1 cm

Shape analysis with boundary directions

Shapes

Roses



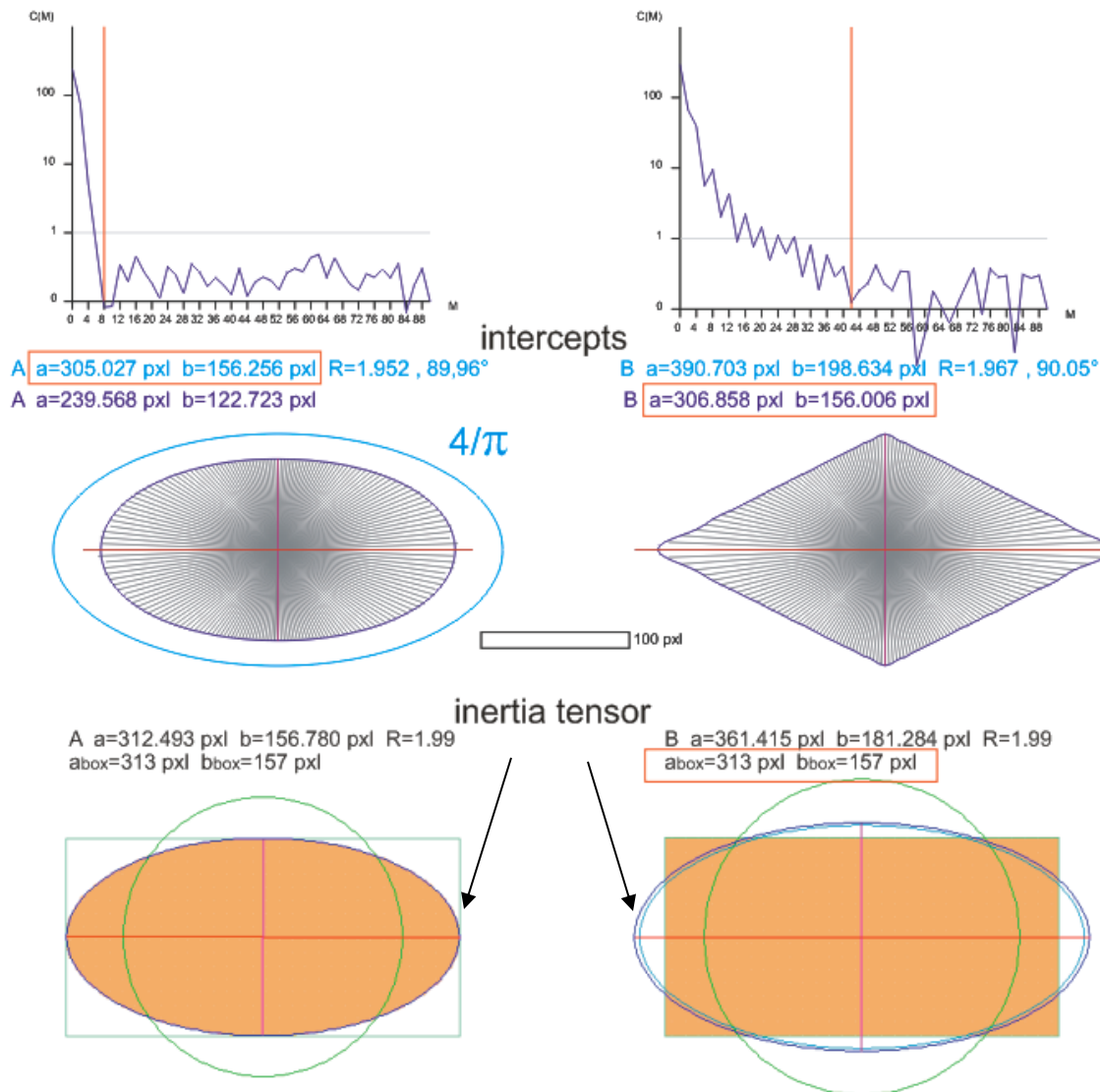
4/π size correction

This is the ratio between the mean intercept length and the long axis of an ellipse

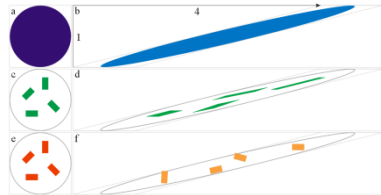
Therefore the correction is :

- valid only for ellipses and population of objects giving an elliptical mean shape

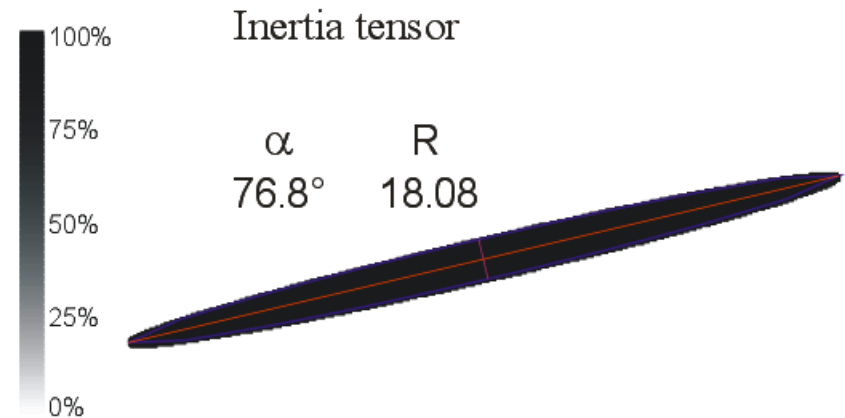
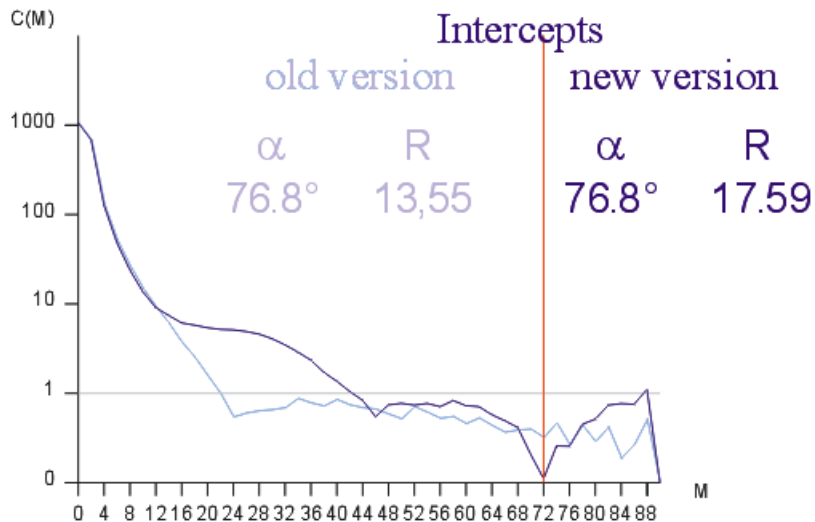
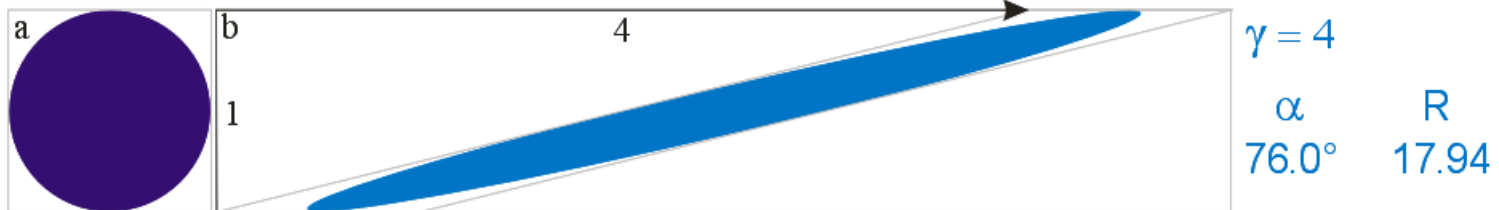
- not valid on rectangles parallel to each other

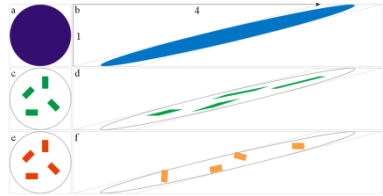


Passive / active deformation of object population



1 passive object



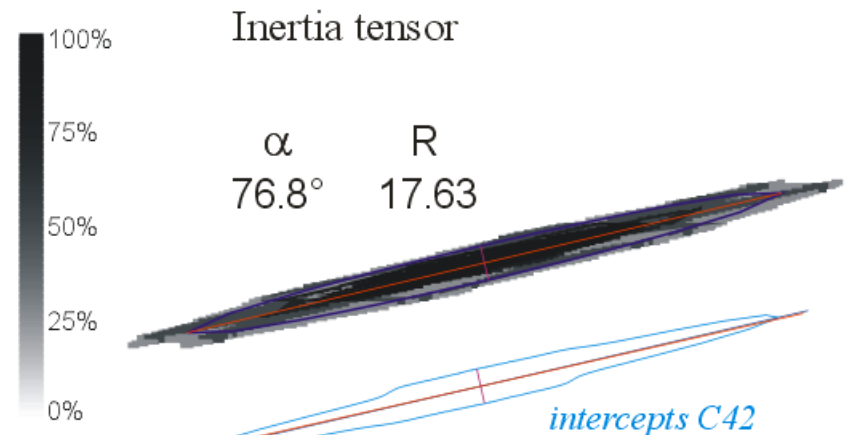
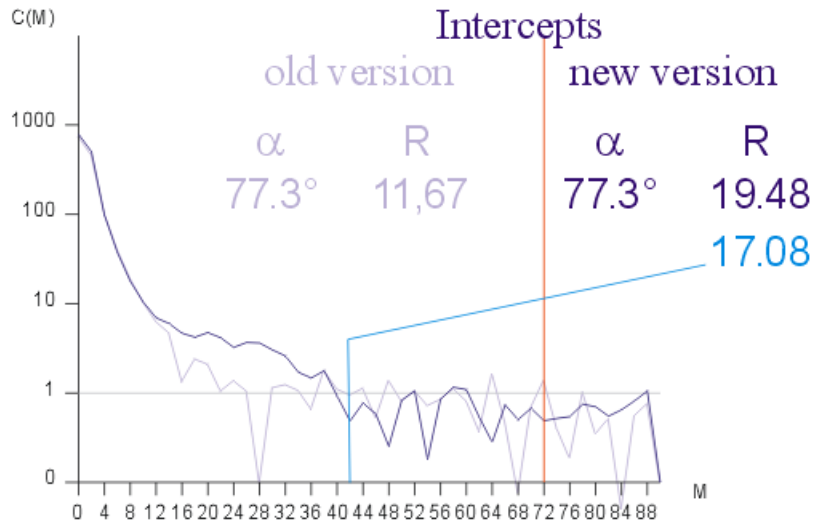


Passive / active deformation of object population

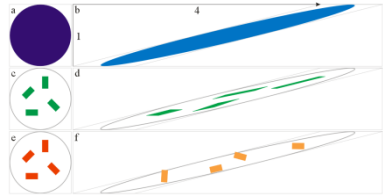
4 passive rectangles



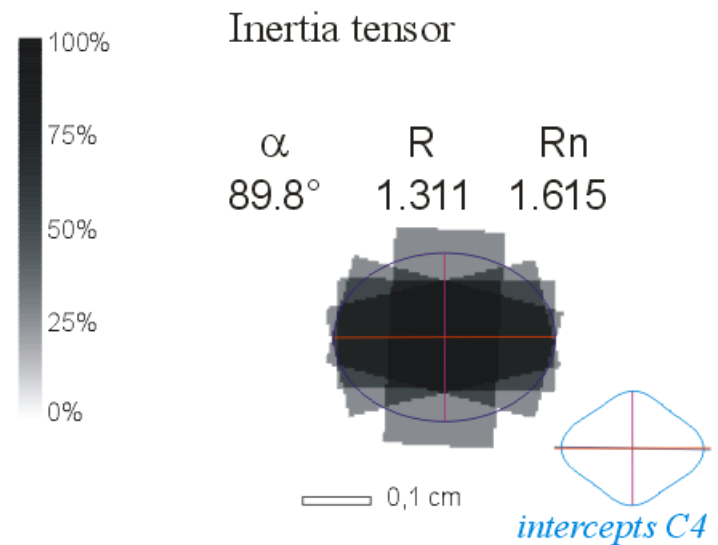
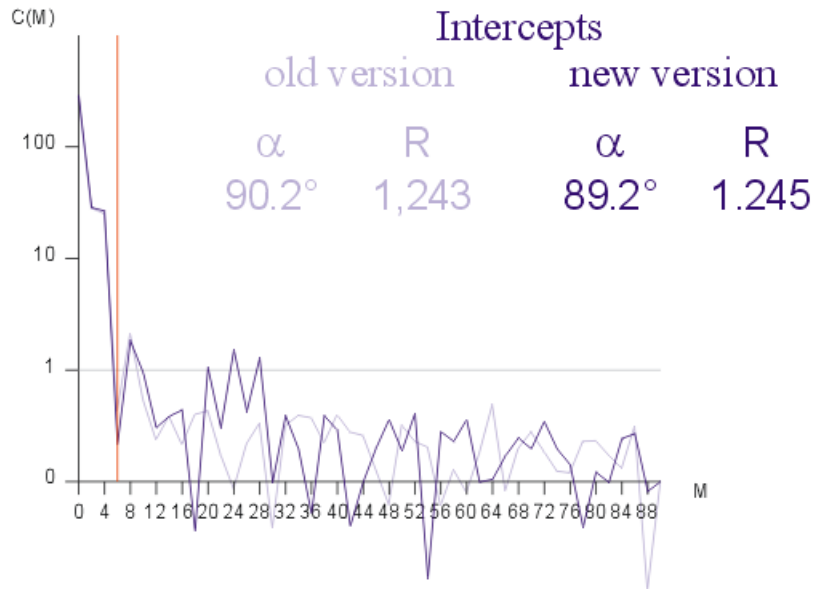
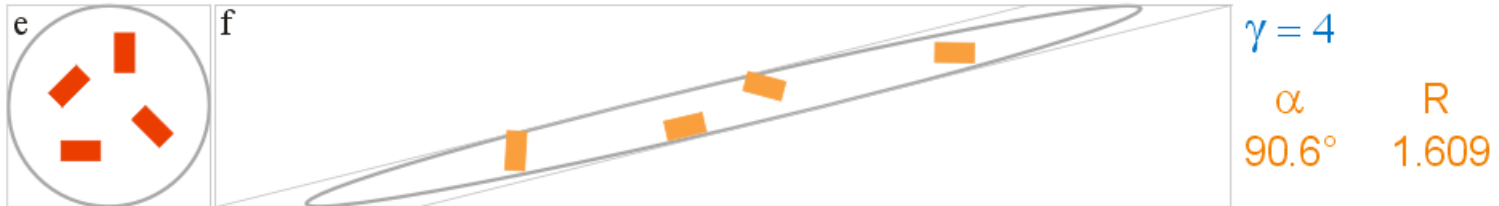
$\gamma = 4$
 $\alpha = 77.4^\circ$ $R = 17.94$



Passive / active deformation of object population



4 active rectangles



A a=2,4855 cm b=1,5591 cm R=1,594 , 40,07°

(1) 39°

Passive deformation of Spirifers

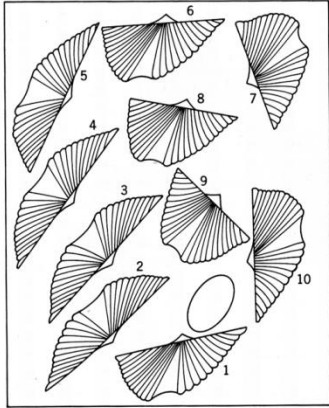


Fig. 5.6. Slab of deformed fossil shells.

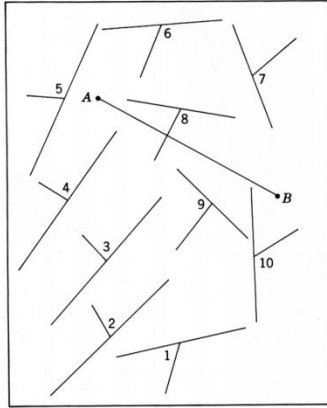


Fig. 5.8. Preliminary steps in construction of strain ellipse by Wellman's method.

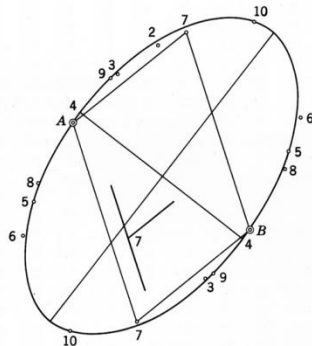
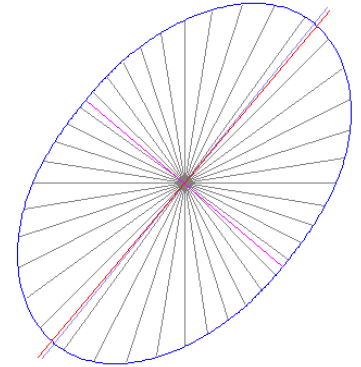
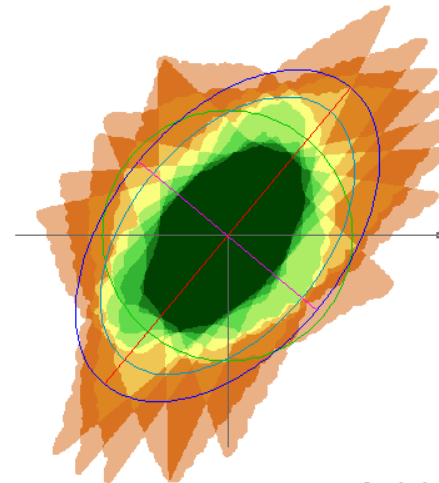
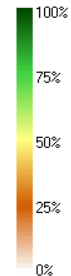


Fig. 5.9. Strain ellipse.

1.59



A n=10 a=3,9526 cm b=2,3792 cm R=1,661 [2,105]n [2,120]b , 39,73° , angle X: 129,73°
K=0,468, Kn=0,632 (0,741), Kbn=0,636 (0,736)



□ 0,1 cm

1.66



1.62

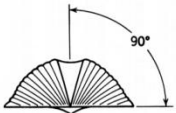
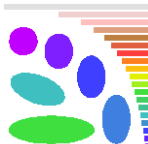


Fig. 5.7. Form of individual shell in undeformed state.

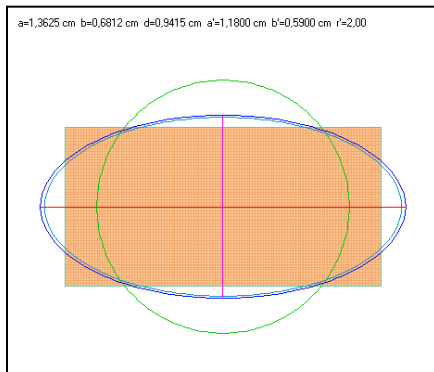


Window of measurement

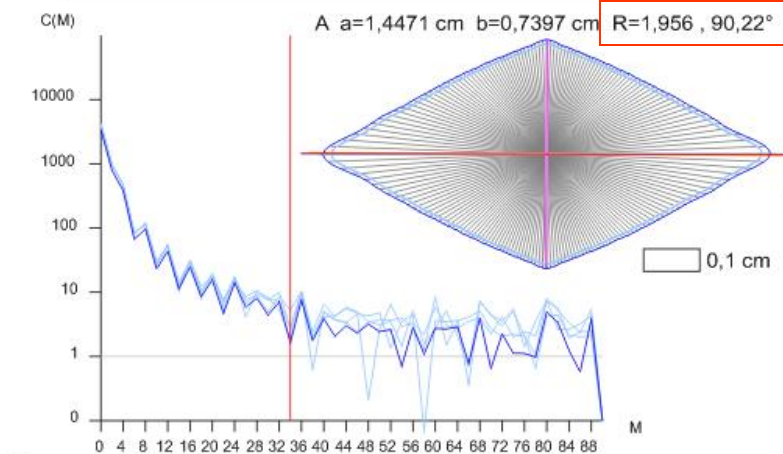
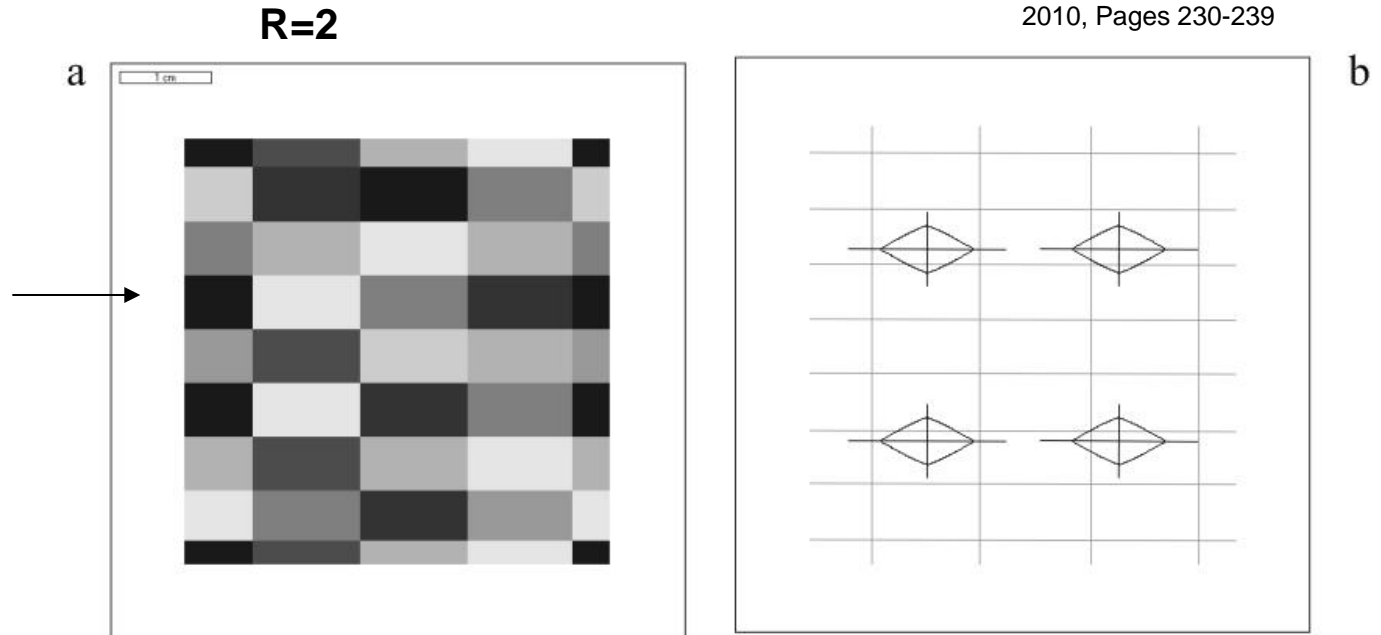
with mask

Means no intercept detection along grey levels of the mask

$a_{box}=1,18\text{cm}$ $b_{box}=0,69\text{cm}$

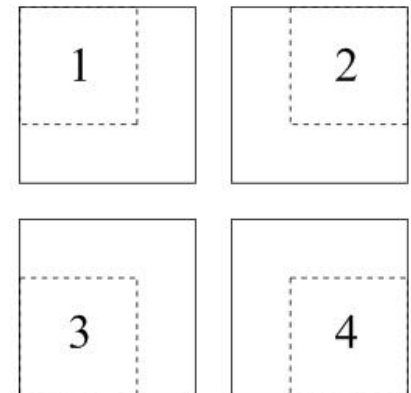


Inertia tensor of one rectangle



c

$a=1.137\text{ cm}$ $b=0.581\text{ cm}$ (without $4/\pi$ correction)



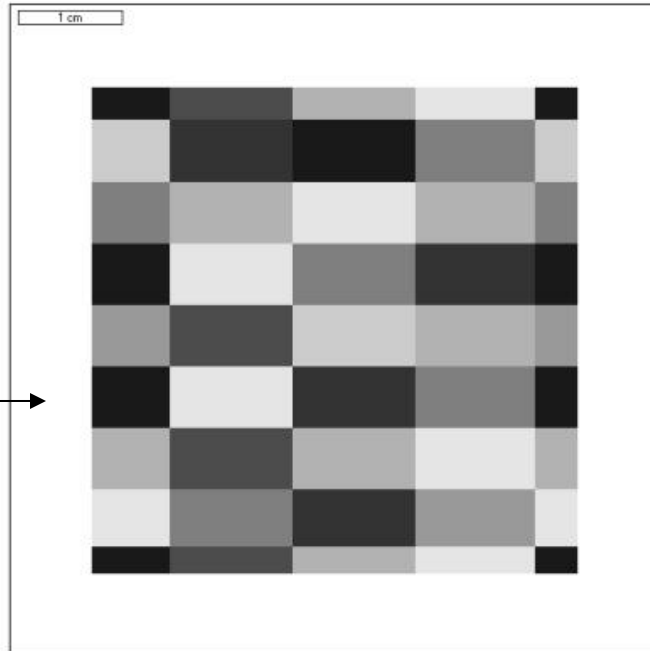
e

Window of measurement

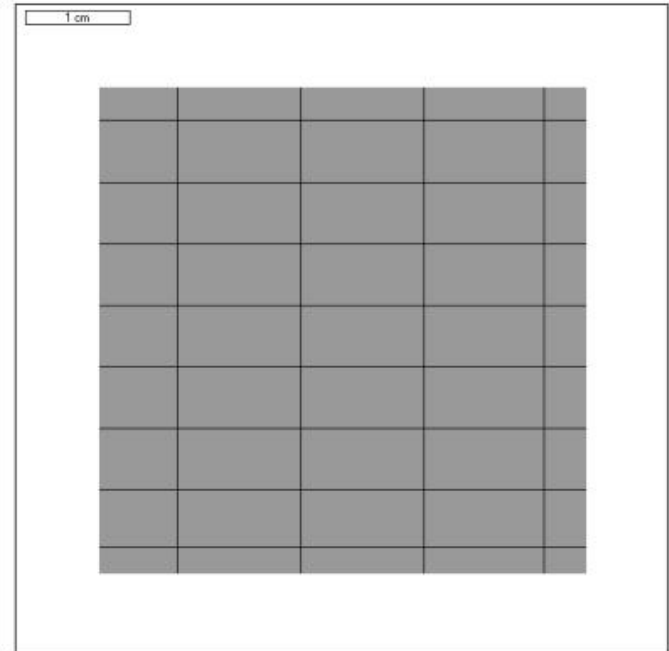
with mask

Means no intercept detection along grey levels of the mask

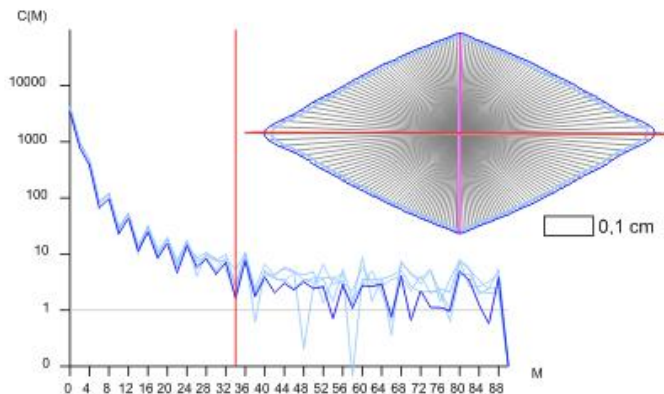
Standard grey levels



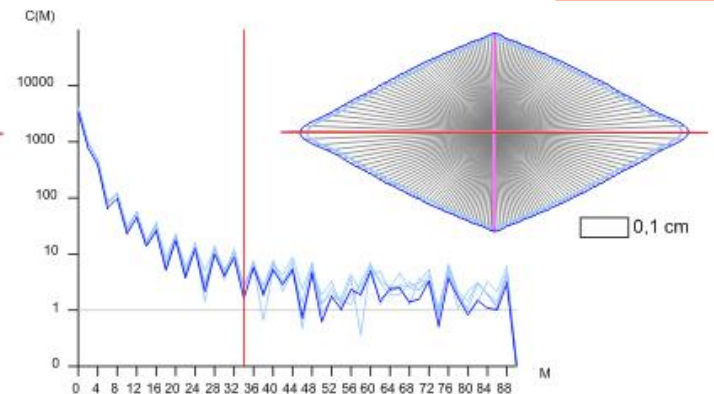
On grey phase with dark edges



A a=1,4471 cm b=0,7397 cm R=1,956 , 90,22°



A a=1,4715 cm b=0,7490 cm R=1,965 , 90,13°



Launeau P., Archanjo C. J., Picard D., Arbaret L., Robin P.Y. (2010). Two- and three-dimensional shape fabric analysis by the intercept method in grey levels. Tectonophysics, Volume 492, Issues 1-4, 20 September 2010, Pages 230-239

abs [-1 1]

if [-1 1] > 0 then display [-1 1]

Direct analysis versus drawing

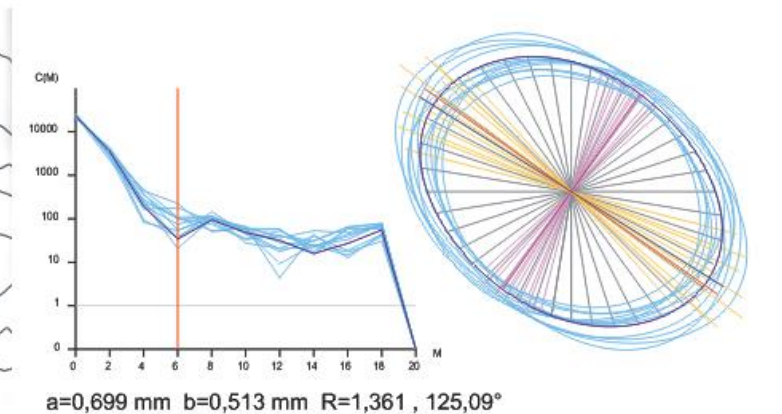
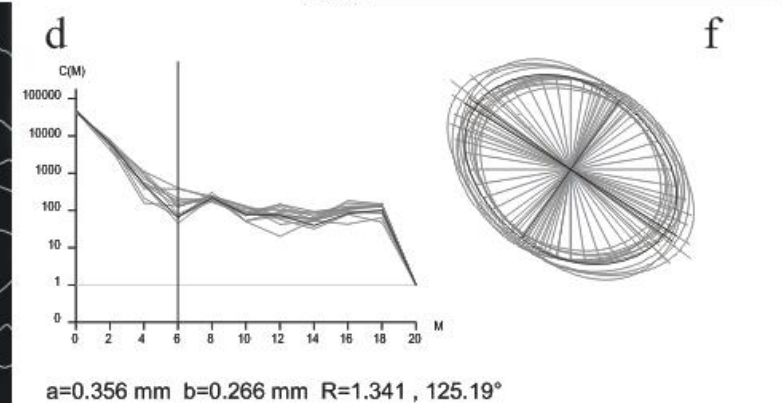
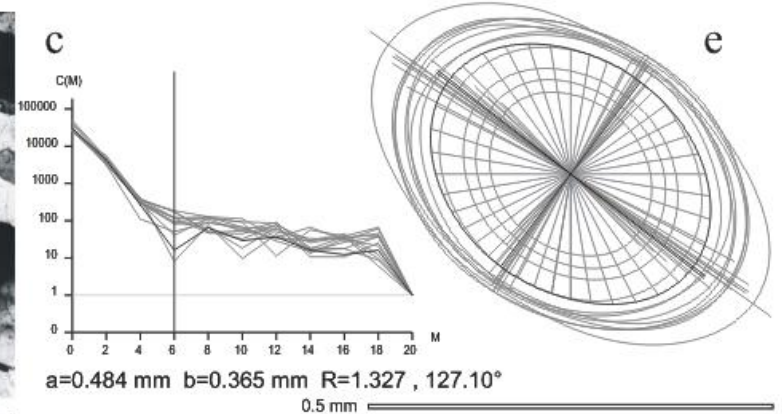
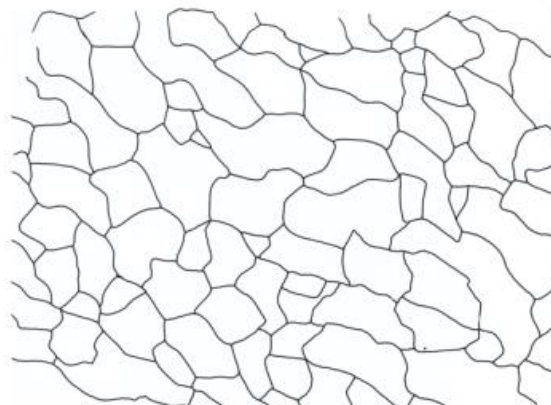
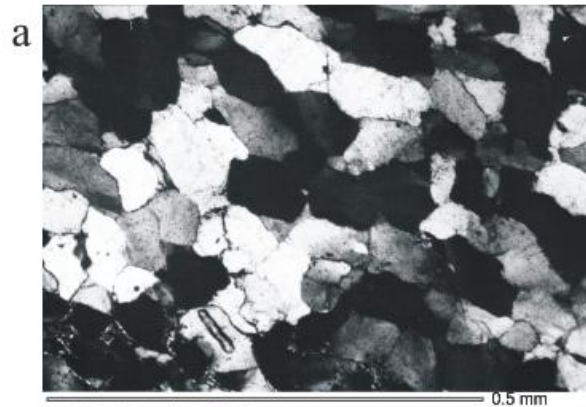
Launeau P., Archanjo C. J., Picard D., Arbaret L., Robin P.Y. (2010). Two- and three-dimensional shape fabric analysis by the intercept method in grey levels. *Tectonophysics*, Volume 492, Issues 1-4, 20 September 2010, Pages 230-239

$abs[-1 \ 1]$

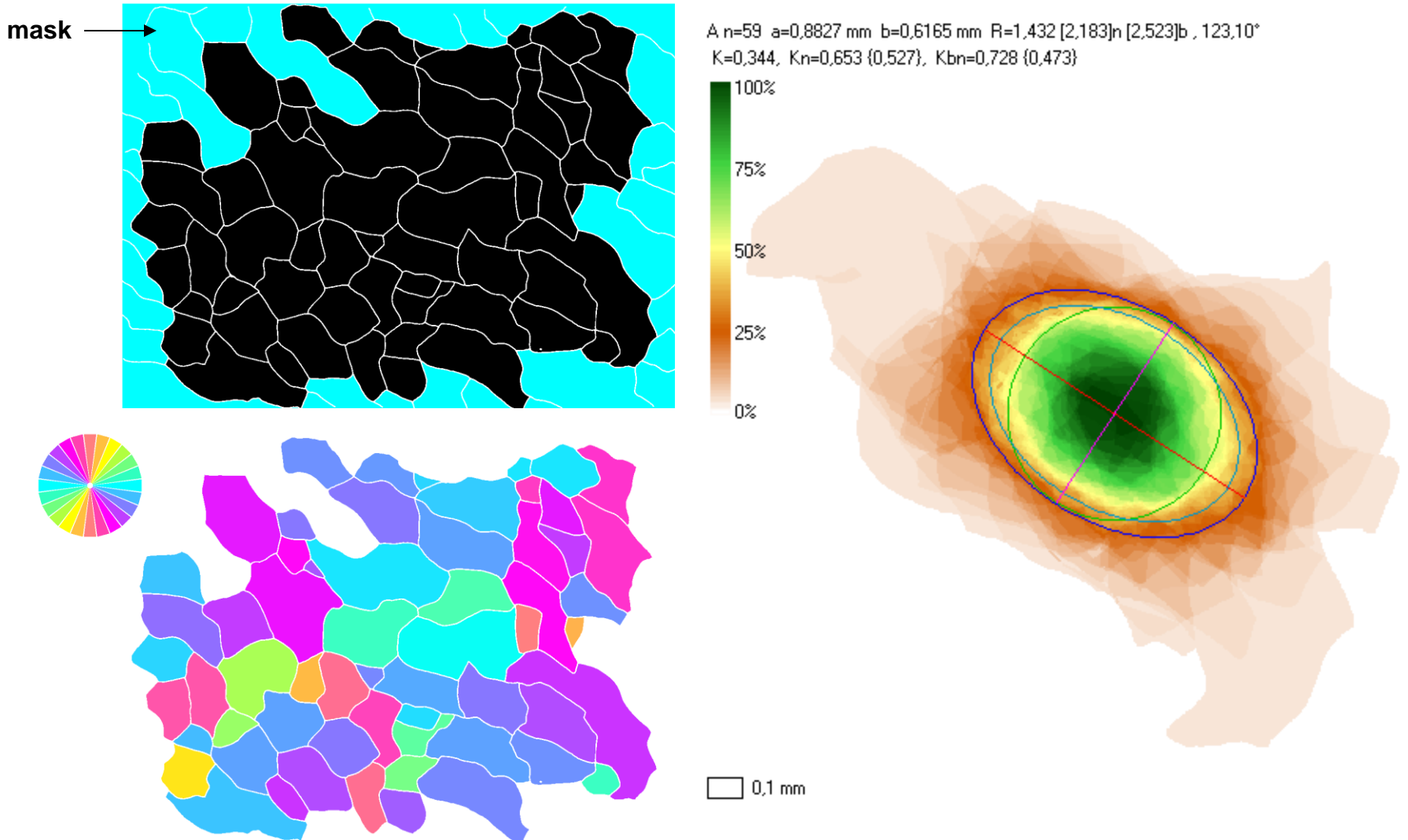
Standard grey levels

if $[-1 \ 1] > 0$ then display $[-1 \ 1]$

One grey phase with dark edges

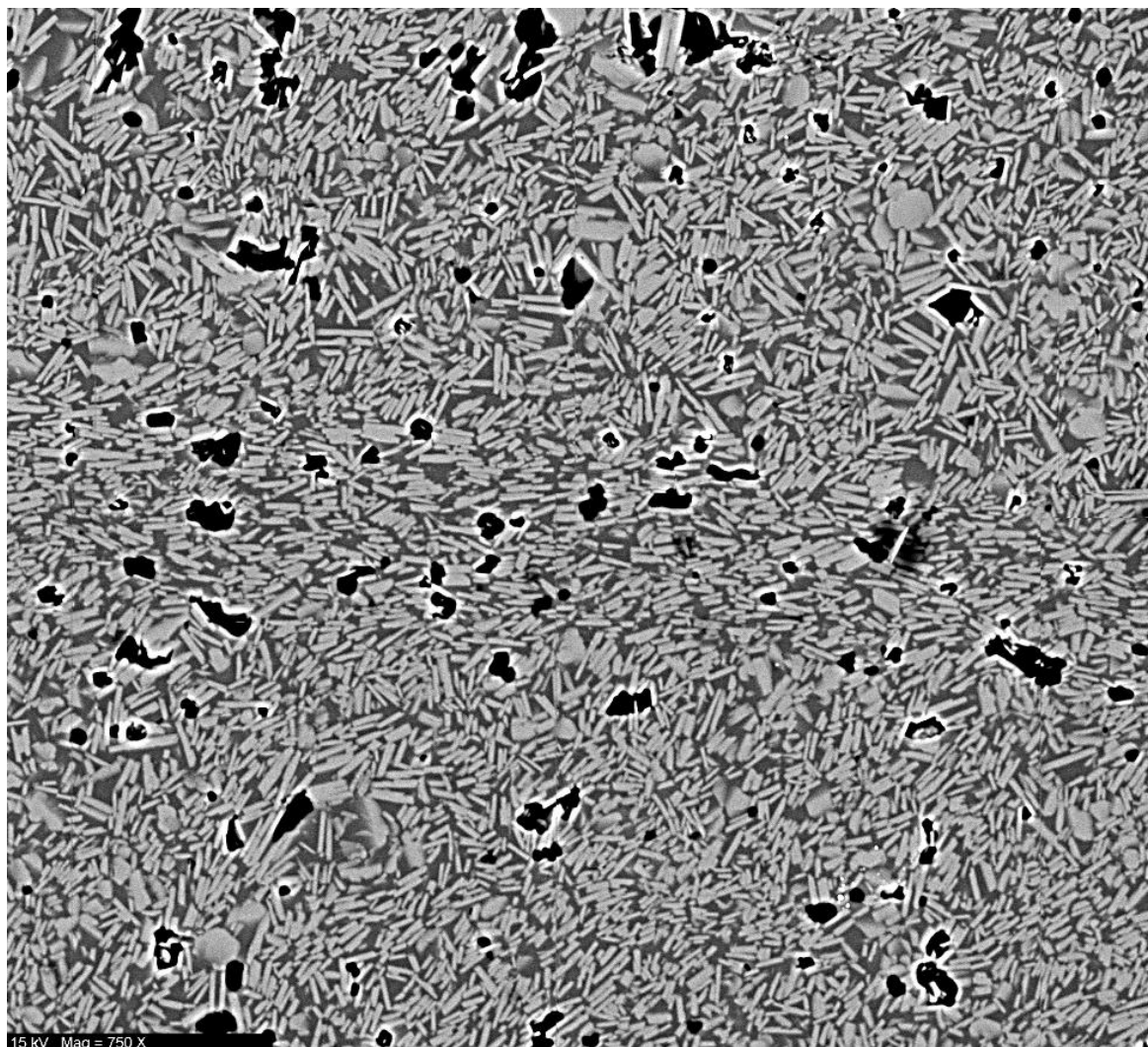


Direct analysis versus drawing with inertia tensor



Application to the BSE image of a synthetic magma

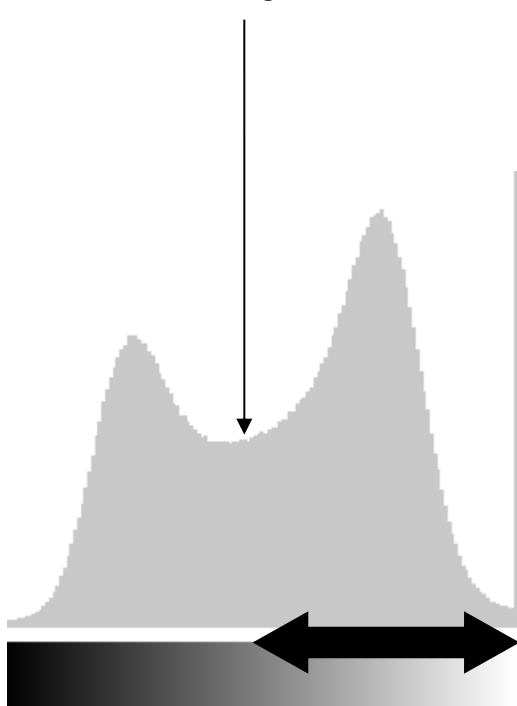
Plagioclase-bearing suspension composed of 52% of crystals was synthesized and then deformed using a Paterson HP-HT apparatus at a confining pressure of 300 MPa, a temperature of 850 C and a shear strain $\gamma = 3.5$



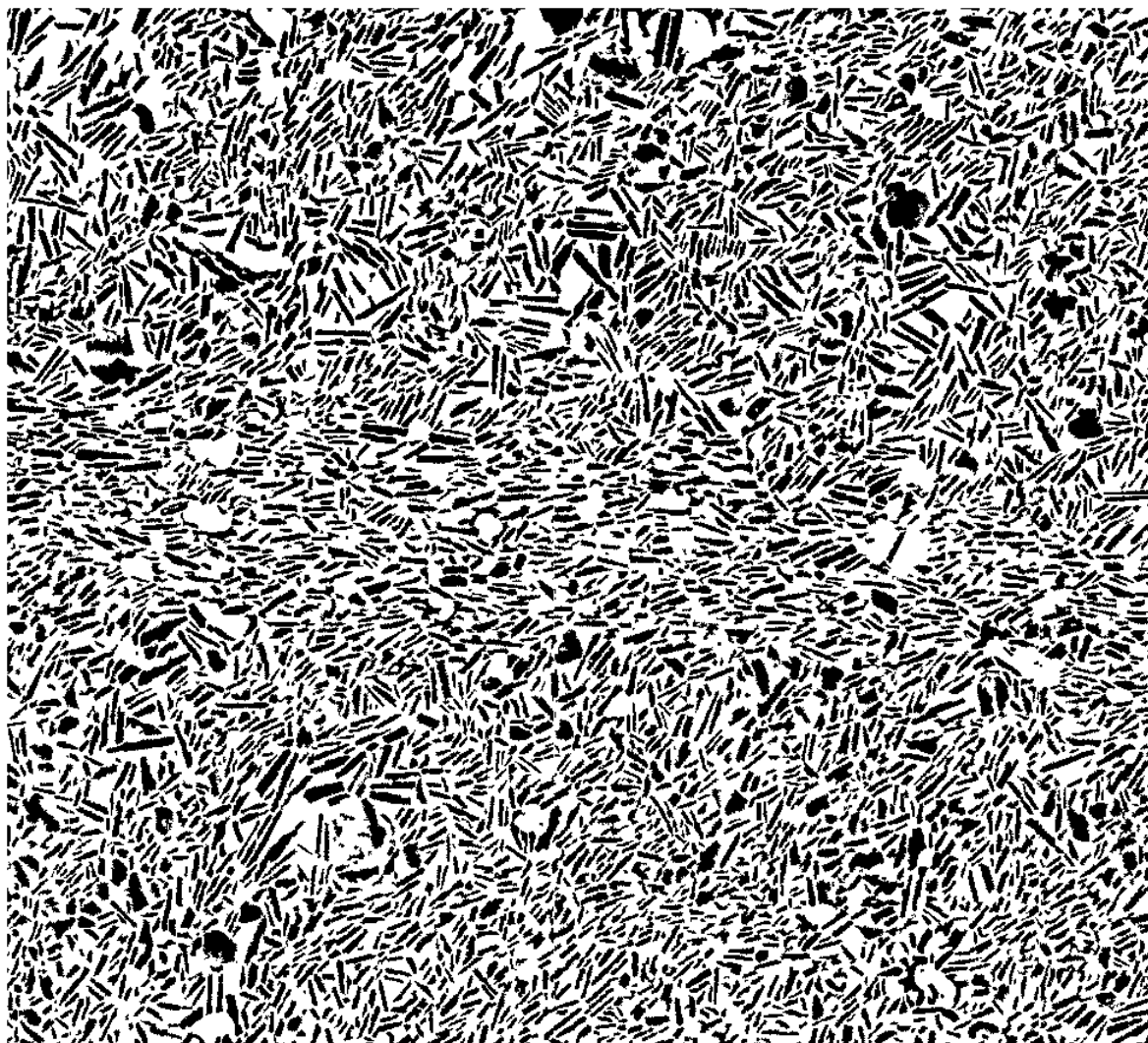
Application to the BSE image of a synthetic magma

Threshold at grey level

140

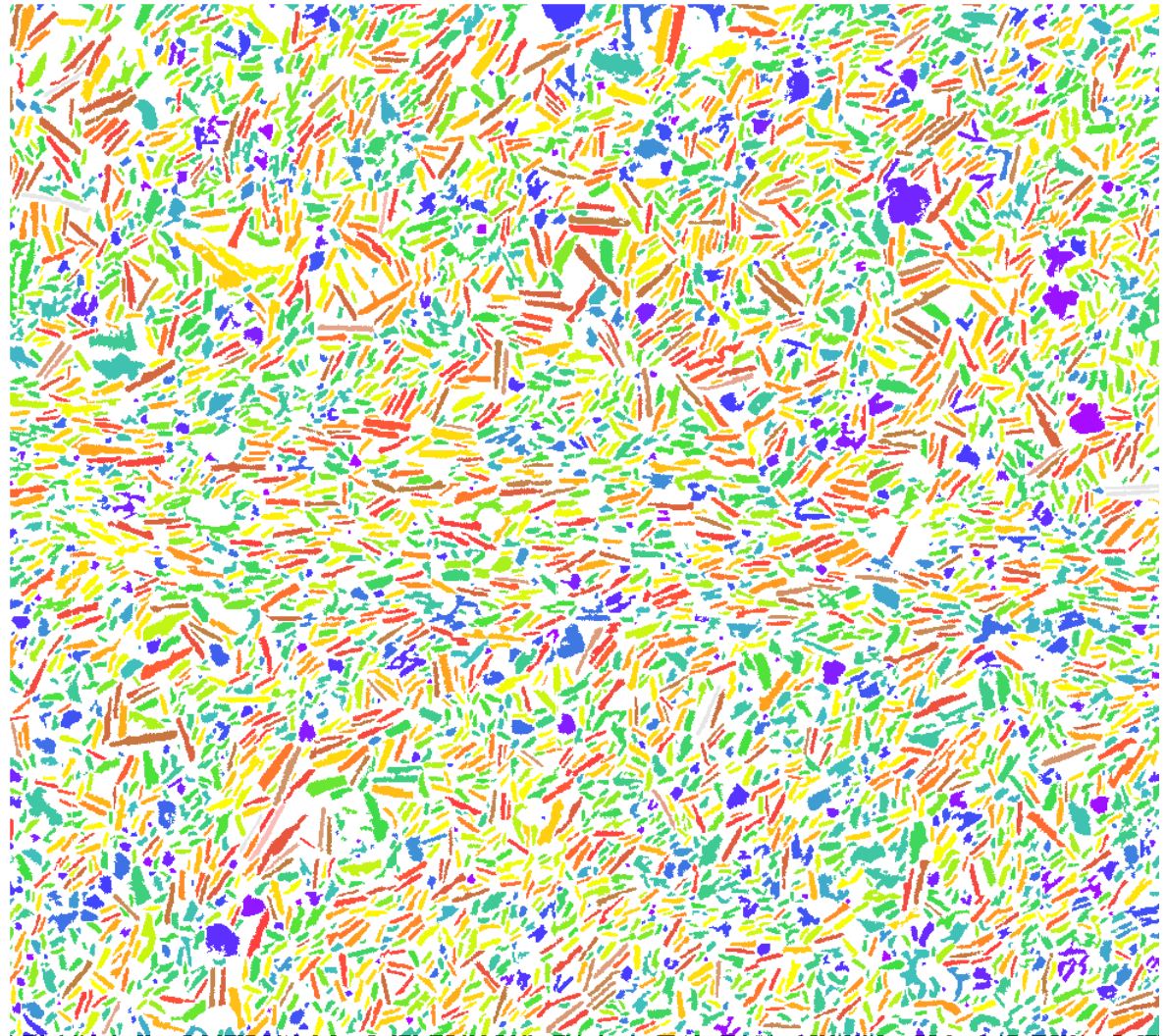
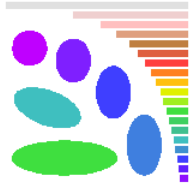


+ drawing a lot of
missing boundaries

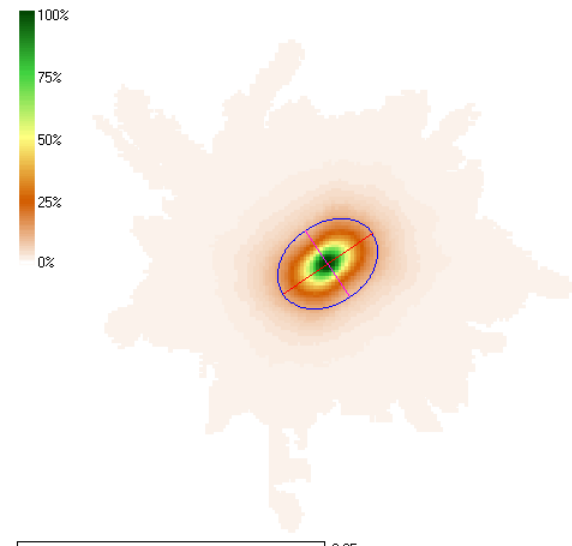


Application to the BSE image of a synthetic magma

Inertia tensor : R

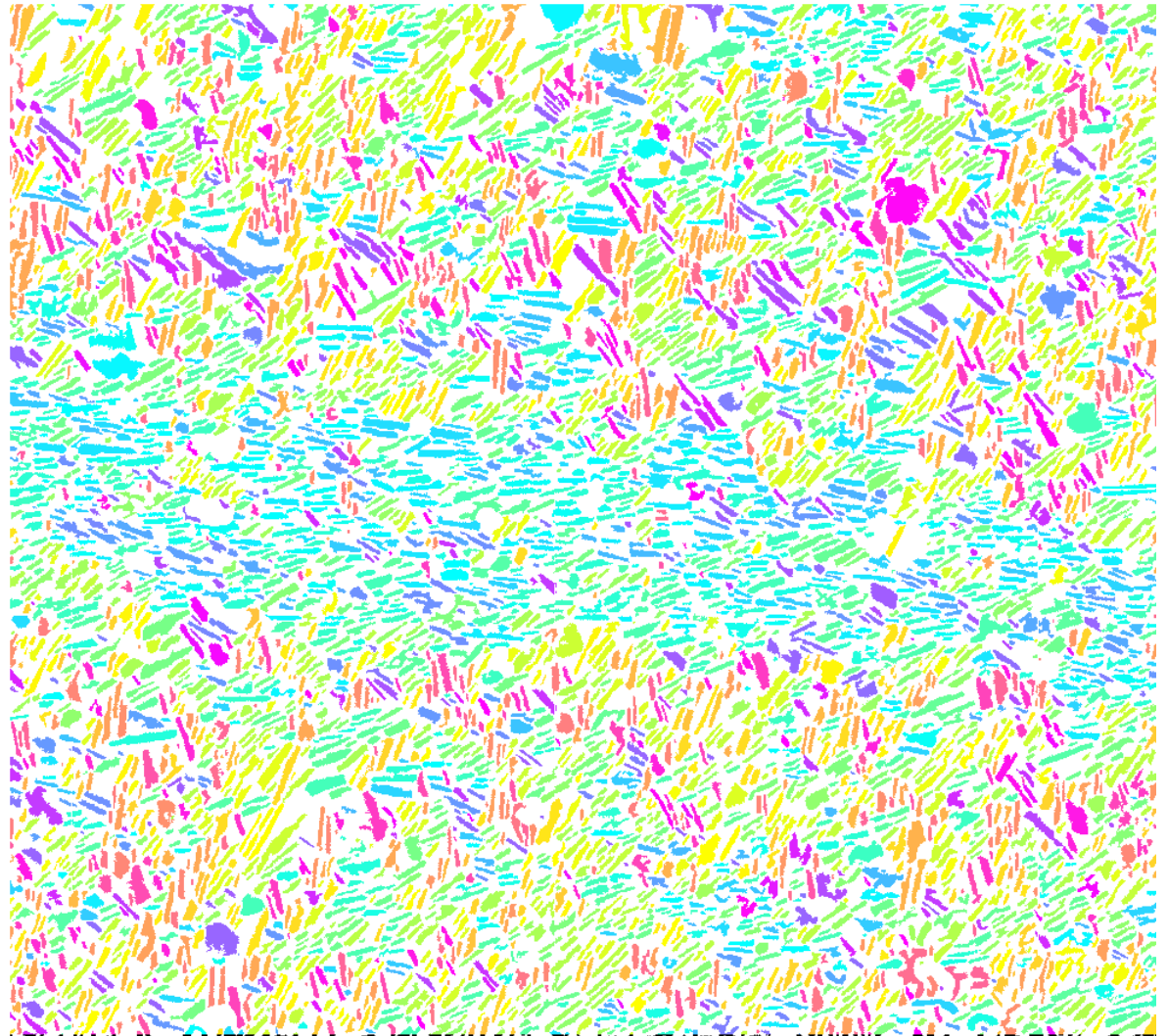


$A_n=4973$ $a=0,0176$ cm $b=0,0131$ cm $R=1,346$ [1,419] η [1,464] ρ , 55,51°
 $K=0,288$, $K_n=0,336$ (0,858), $K_{bn}=0,364$ (0,793)

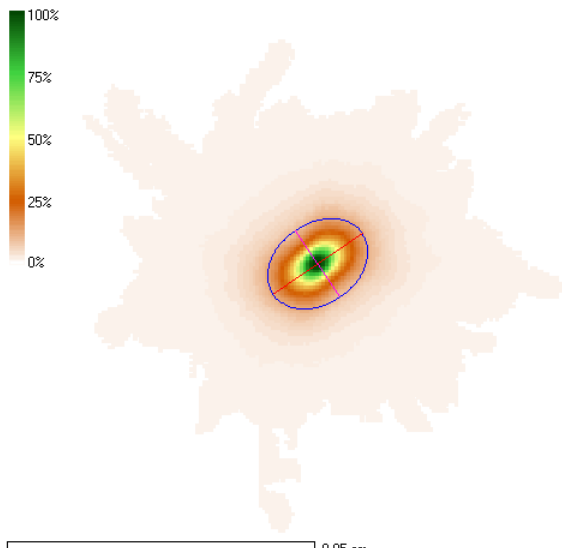


Application to the BSE image of a synthetic magma

Inertia tensor : α



A n=4973 a=0,0176 cm b=0,0131 cm R=1,346 [1,419]n [1,464]b , 55,51°
K=0,288, Kn=0,336 (0,858), Kbn=0,364 (0,793)

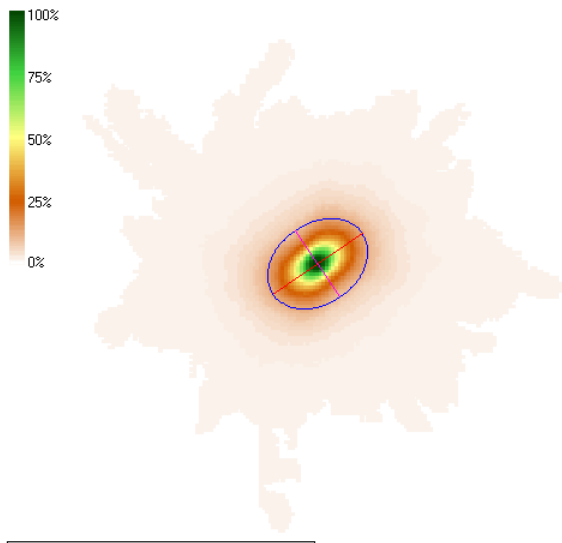


Application to the BSE image of a synthetic magma

Inertia tensor : R, α

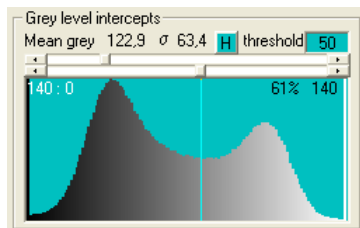


$A n=4973$ $a=0,0176$ cm $b=0,0131$ cm $R=1,346$ [1,419] n [1,464] b , 55,51°
 $K=0,288$, $K_n=0,336$ (0,858), $K_{bn}=0,364$ (0,793)

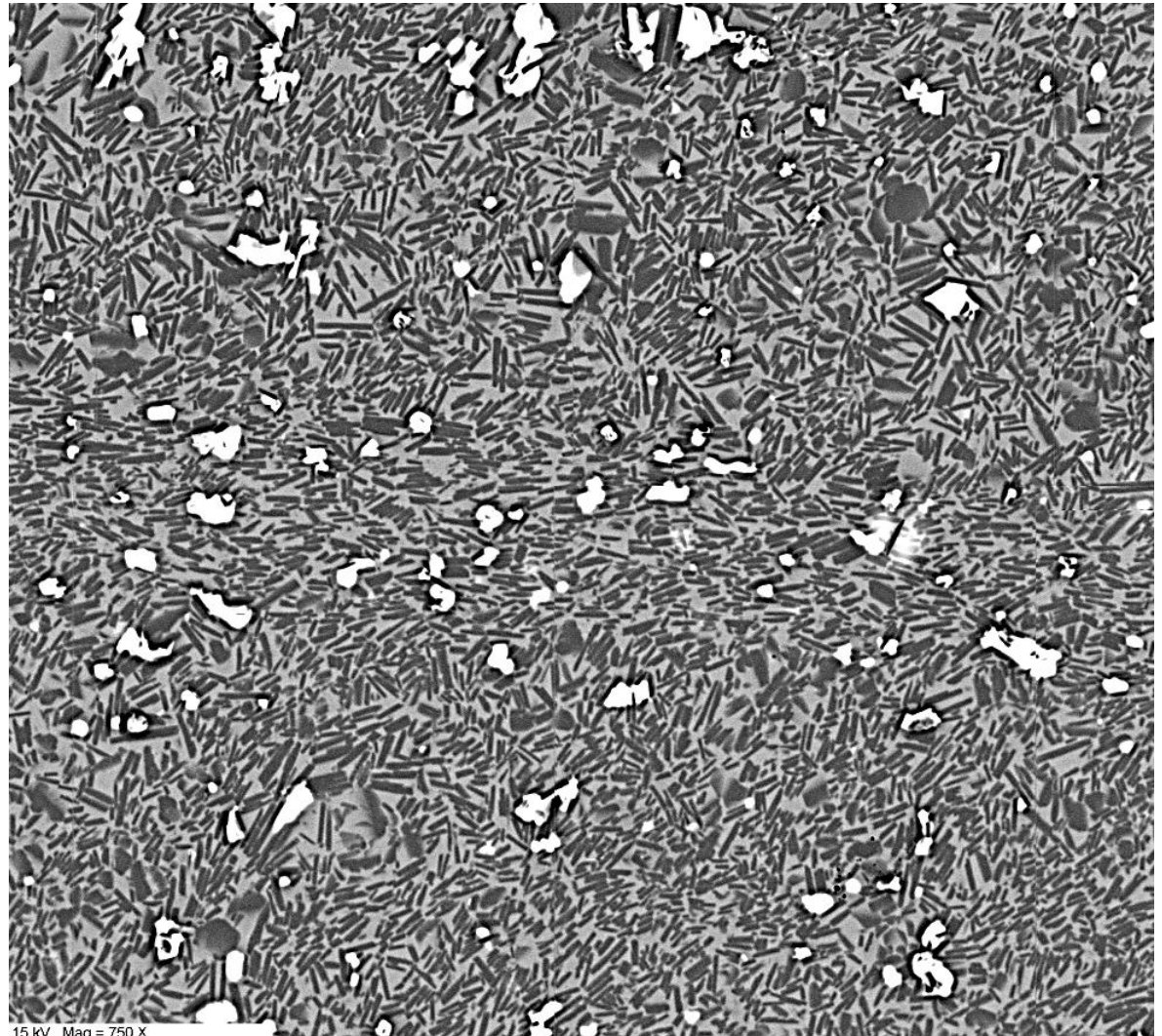


Application to the BSE image of a synthetic magma

Analysis by intercept in
grey level

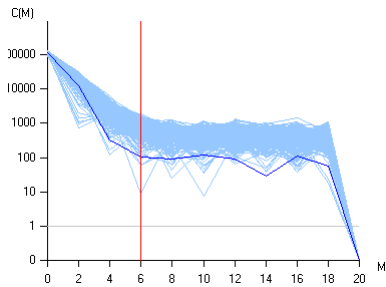


boundary = \longleftrightarrow



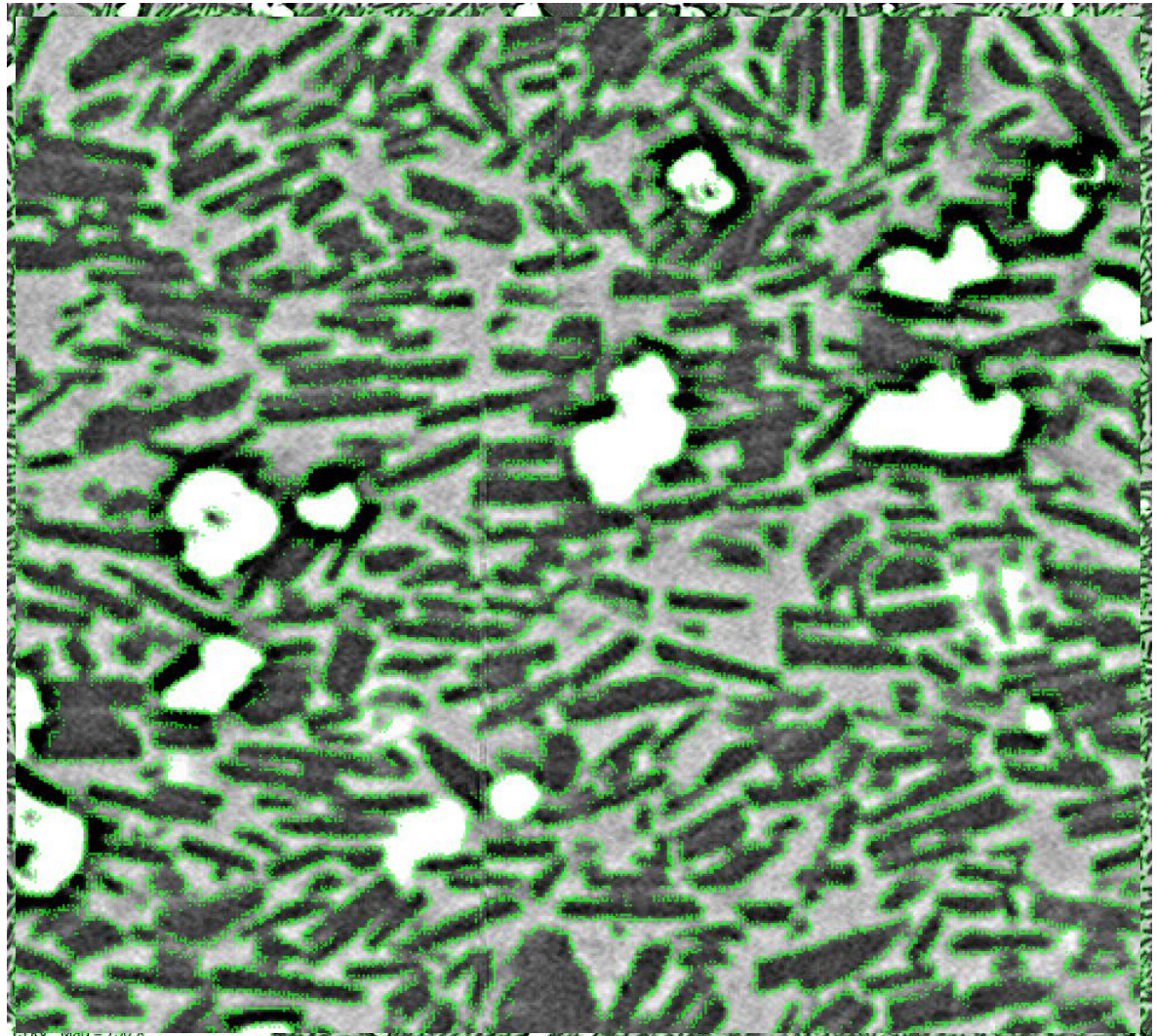
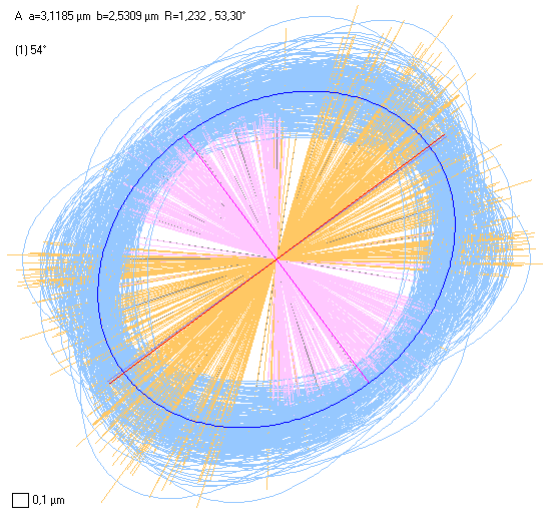
Application to the BSE image of a synthetic magma

Analysis by intercept in grey level



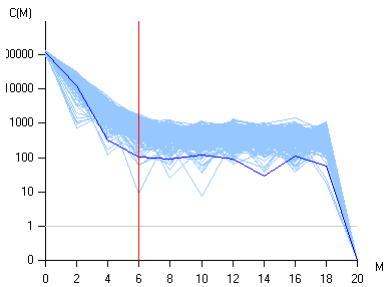
A $a=3,1185 \mu\text{m}$ $b=2,5309 \mu\text{m}$ $R=1,232, 53,30^\circ$

(1) 54°



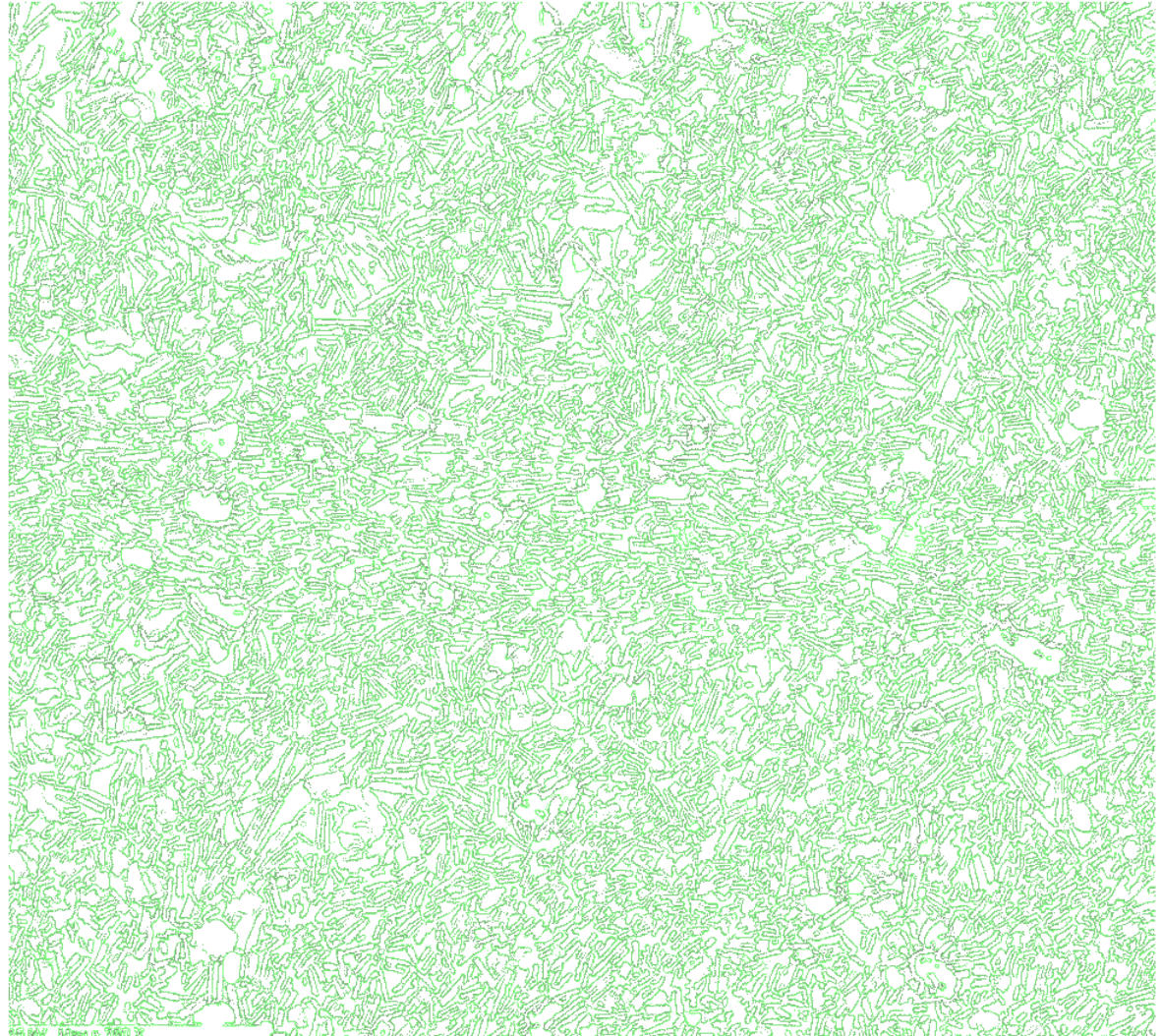
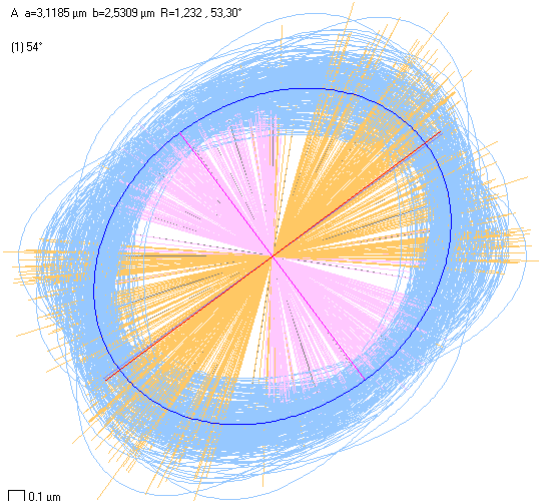
Application to the BSE image of a synthetic magma

Analysis by intercept in grey level



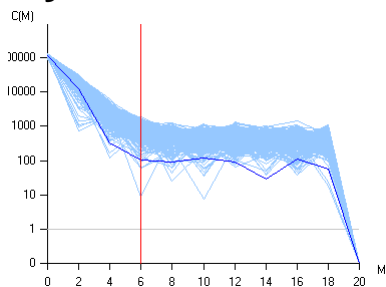
A $a=3,1185 \mu\text{m}$ $b=2,5309 \mu\text{m}$ $R=1,232, 53,30^\circ$

(1) 54°



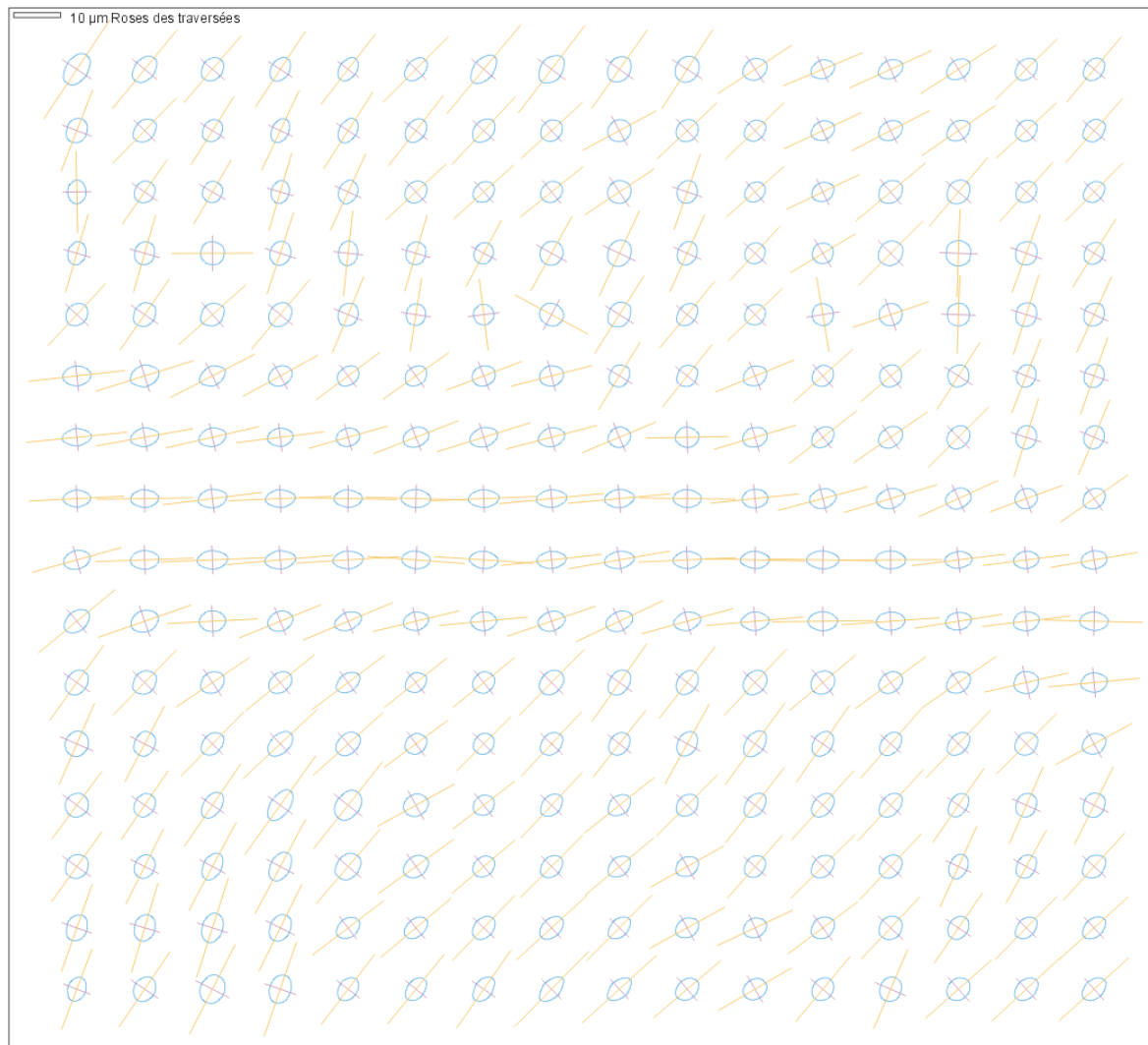
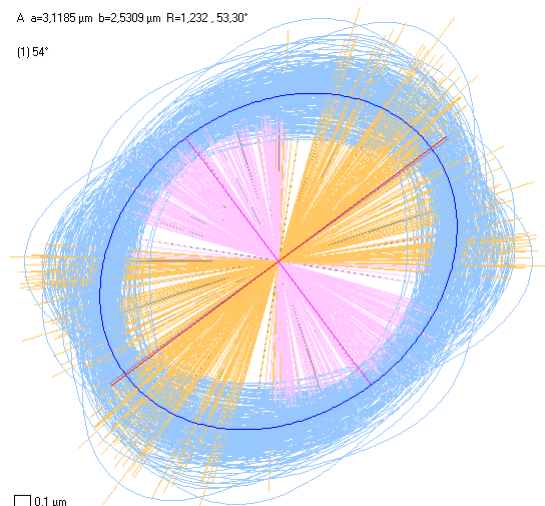
Application to the BSE image of a synthetic magma

Analysis by intercept in grey level



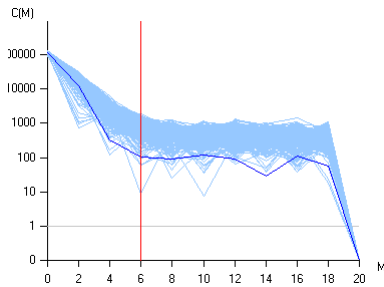
A $a=3,1185 \mu\text{m}$ $b=2,5309 \mu\text{m}$ $R=1,232, 53,30^\circ$

(1) 54°



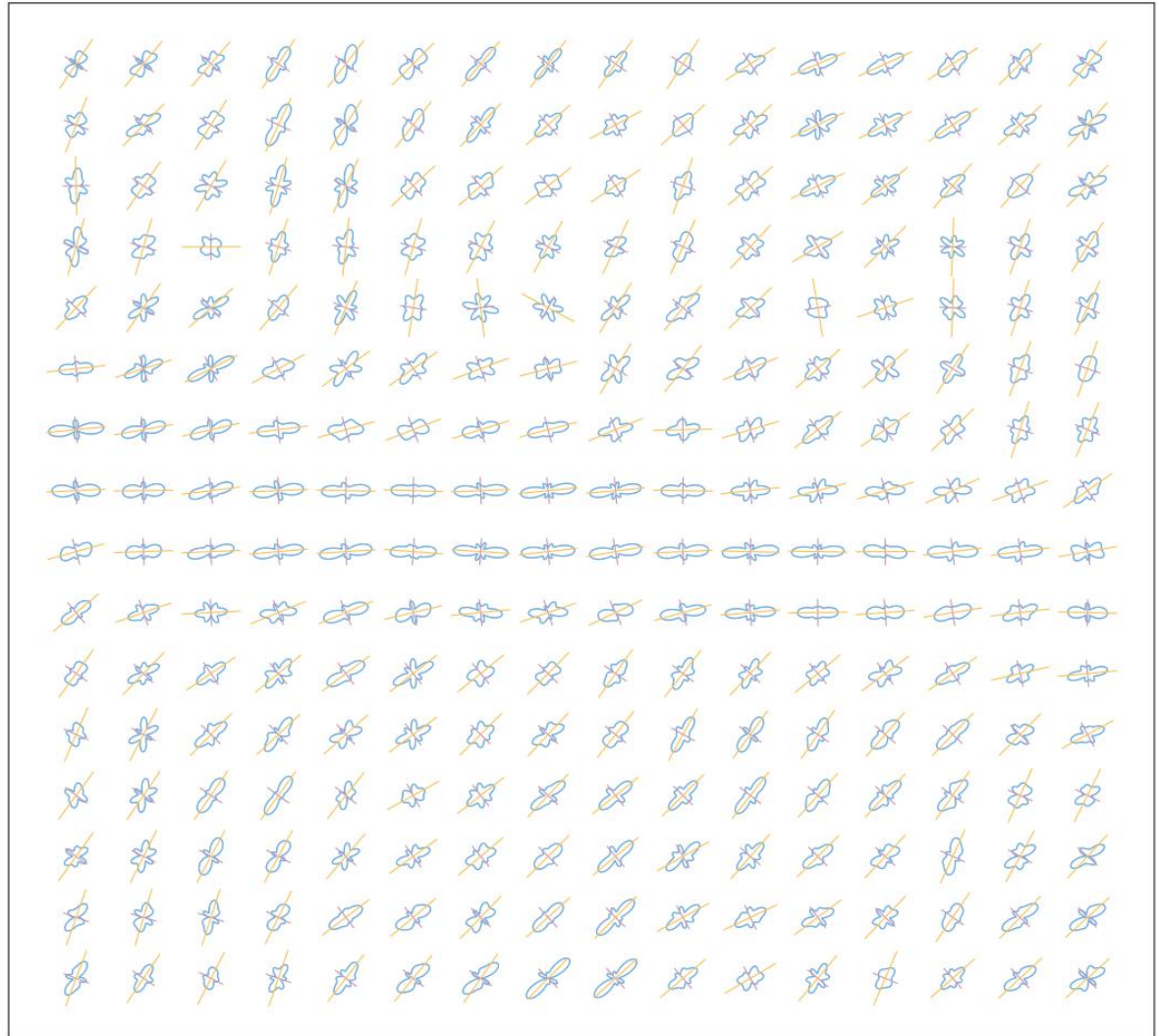
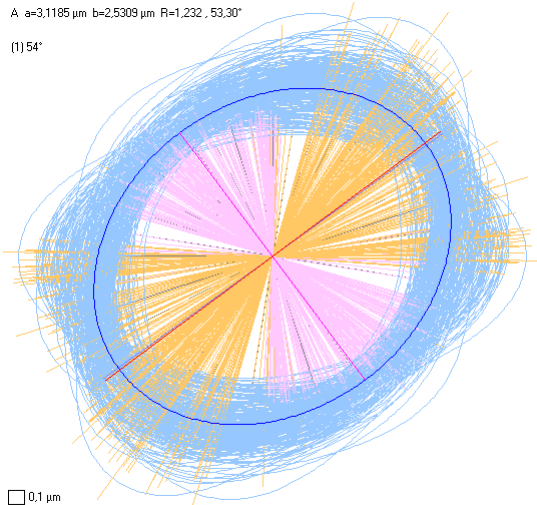
Application to the BSE image of a synthetic magma

Analysis by intercept in grey level



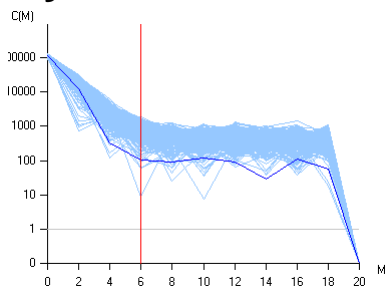
A = 3,1185 μm b = 2,5309 μm R = 1,232, 53,30°

(1) 54°



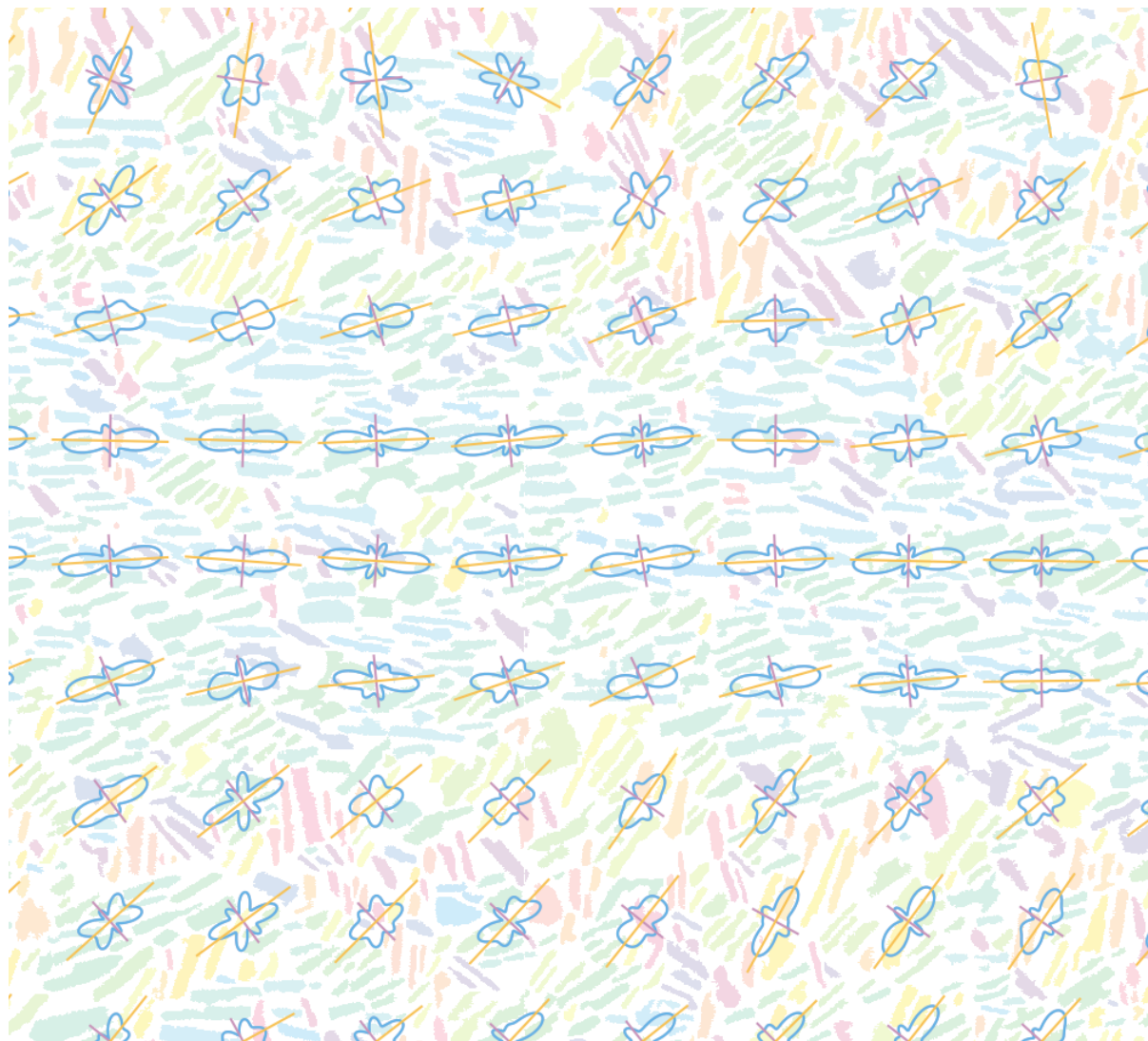
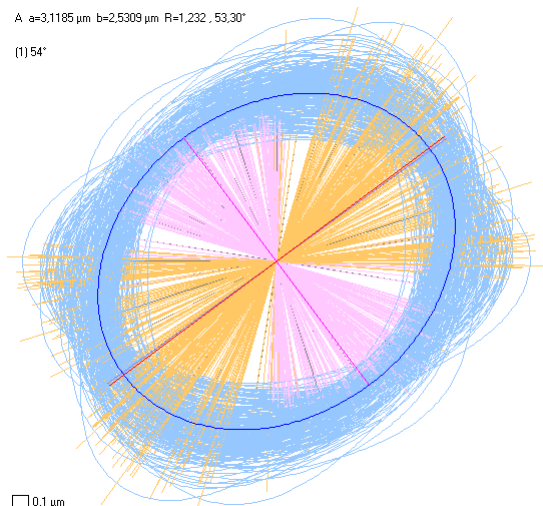
Application to the BSE image of a synthetic magma

Analysis by intercept in grey level



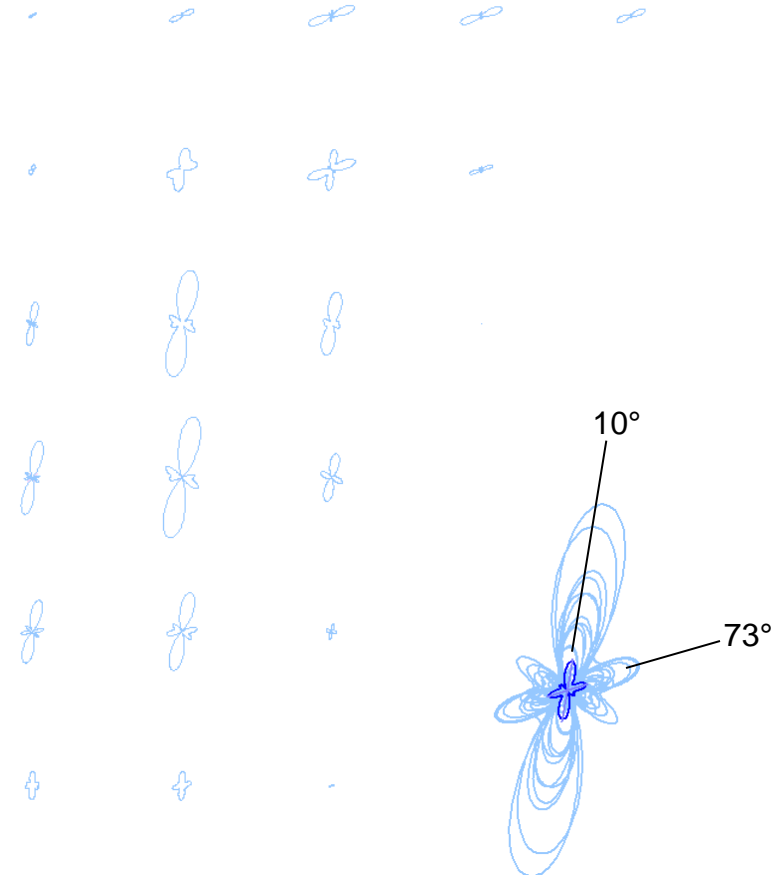
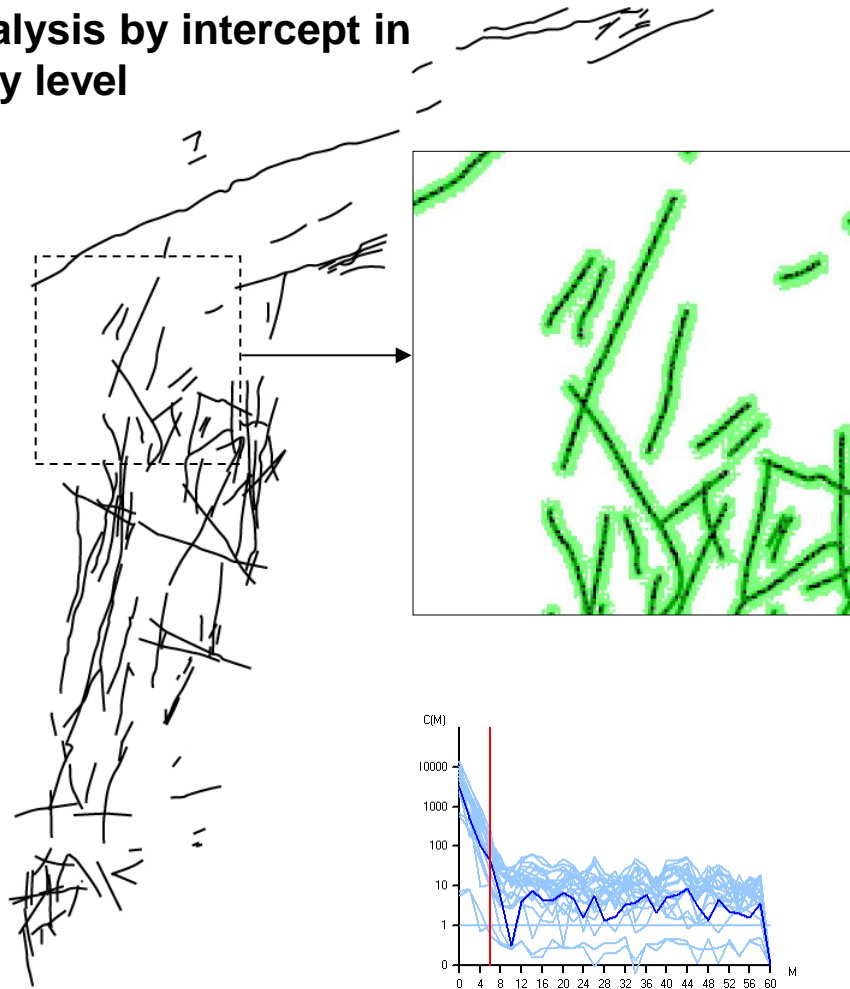
A $a=3,1185 \mu\text{m}$ b $=2,5309 \mu\text{m}$ R $=1,232$, 53,30°

(1) 54°



Application to lineaments (Kaapvaal dikes - South Africa)

Analysis by intercept in
grey level



Patrick Launeau

Quantitative Image Analysis of Minerals and Rocks

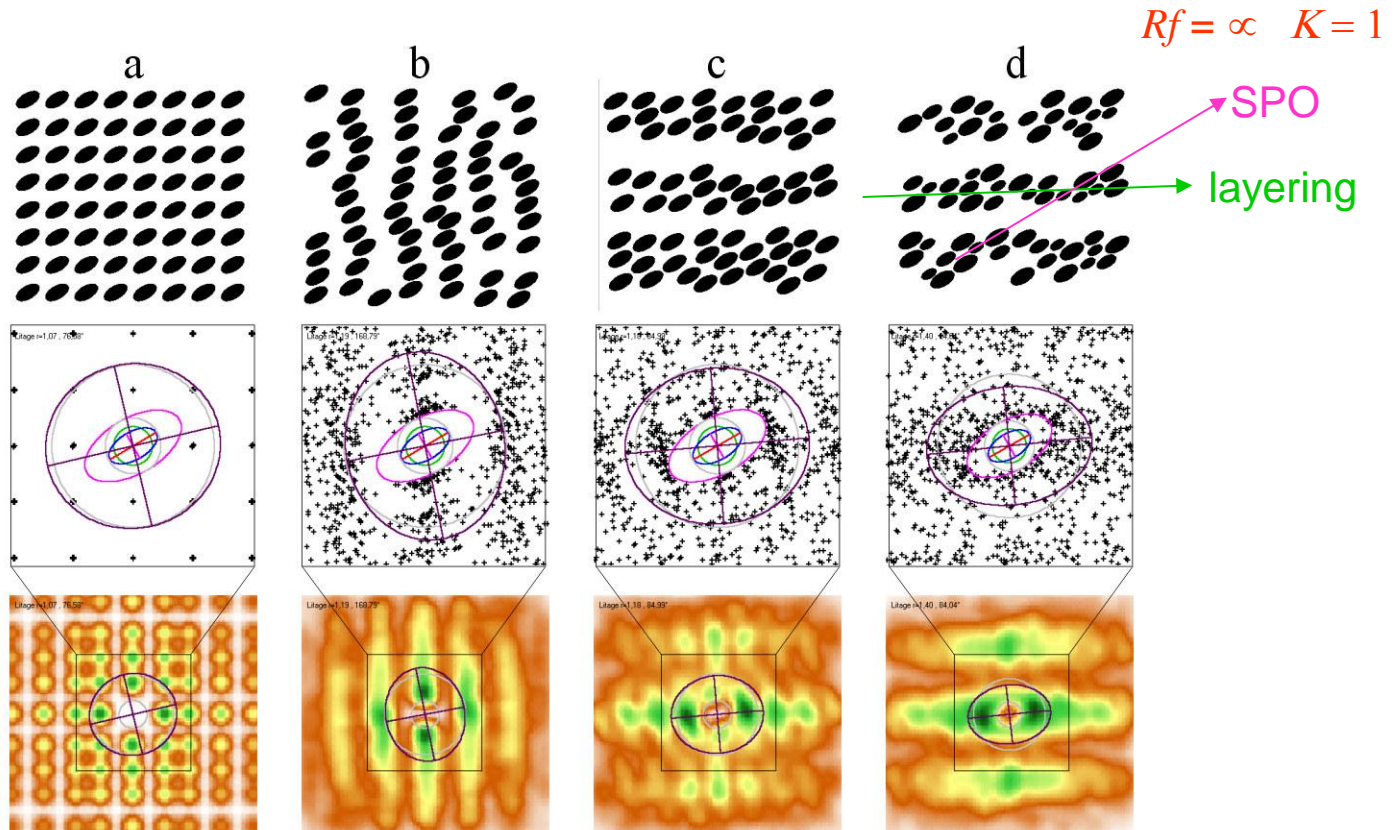
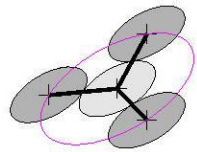
Fabric analysis

- Shape preferred orientation (SPO) vs. strain quantification.
- Intercepts in digital images : a tool to analyze interconnection of grains in rocks vs. inertia tensor of individualized grains
- SPO vs Spatial distribution (Fry)
- Ellipsoid of SPO and strain by combining 3 \perp images.

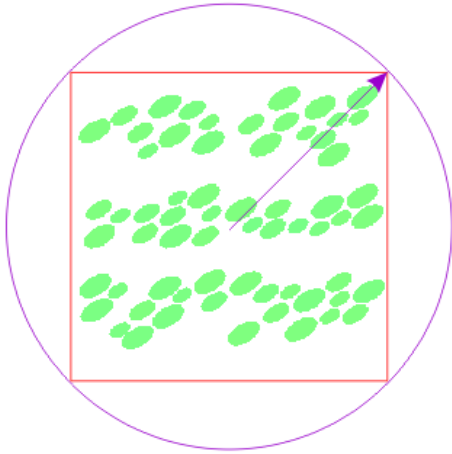


Inertia tensor = Fry diagram

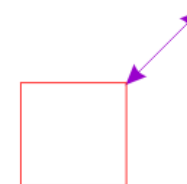
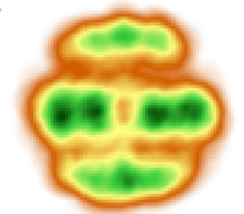
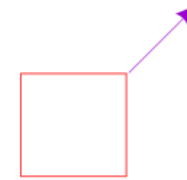
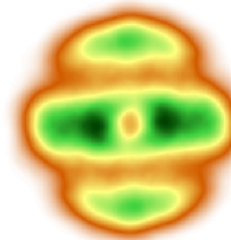
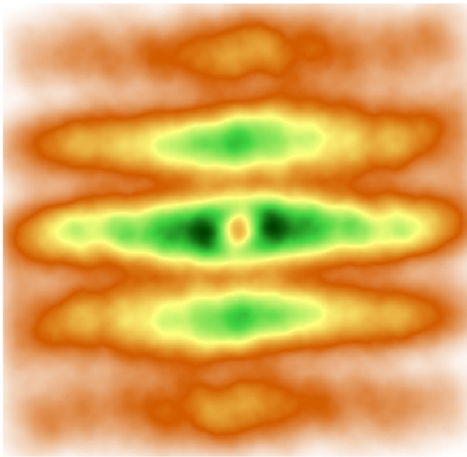
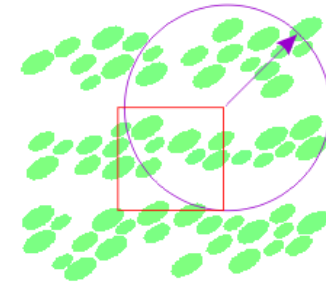
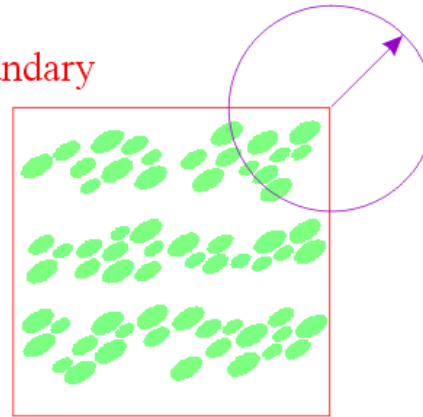
Centre to centre method = Spatial distribution



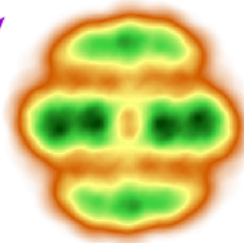
minimum distance



boundary



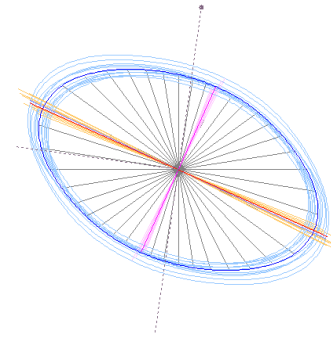
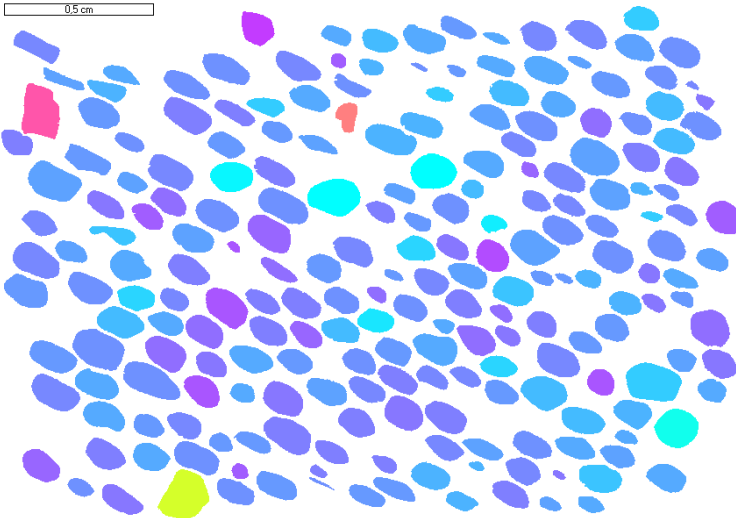
+ symmetry



sensitive to boundary

not
sensitive to boundary

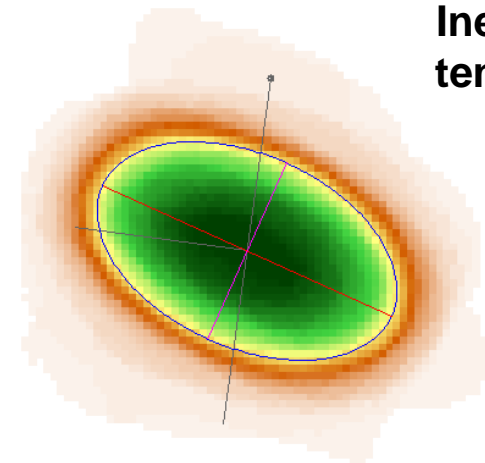
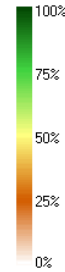
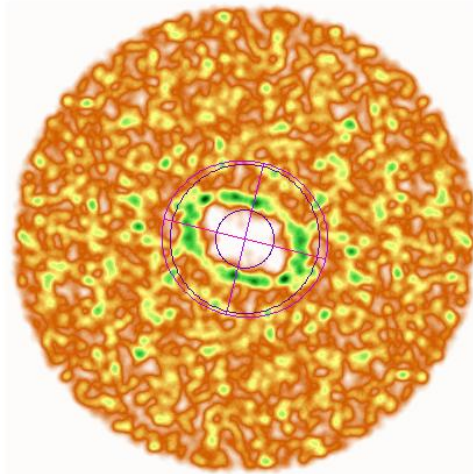
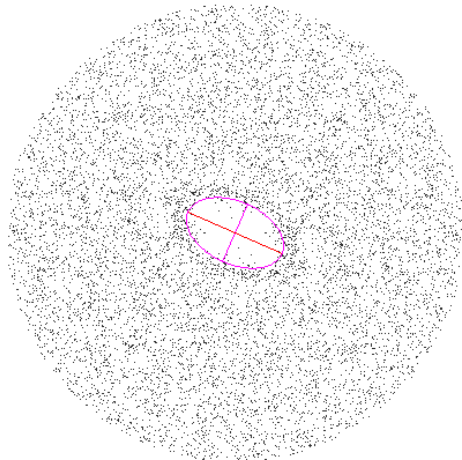
Fry



**Mean
intercept
length**

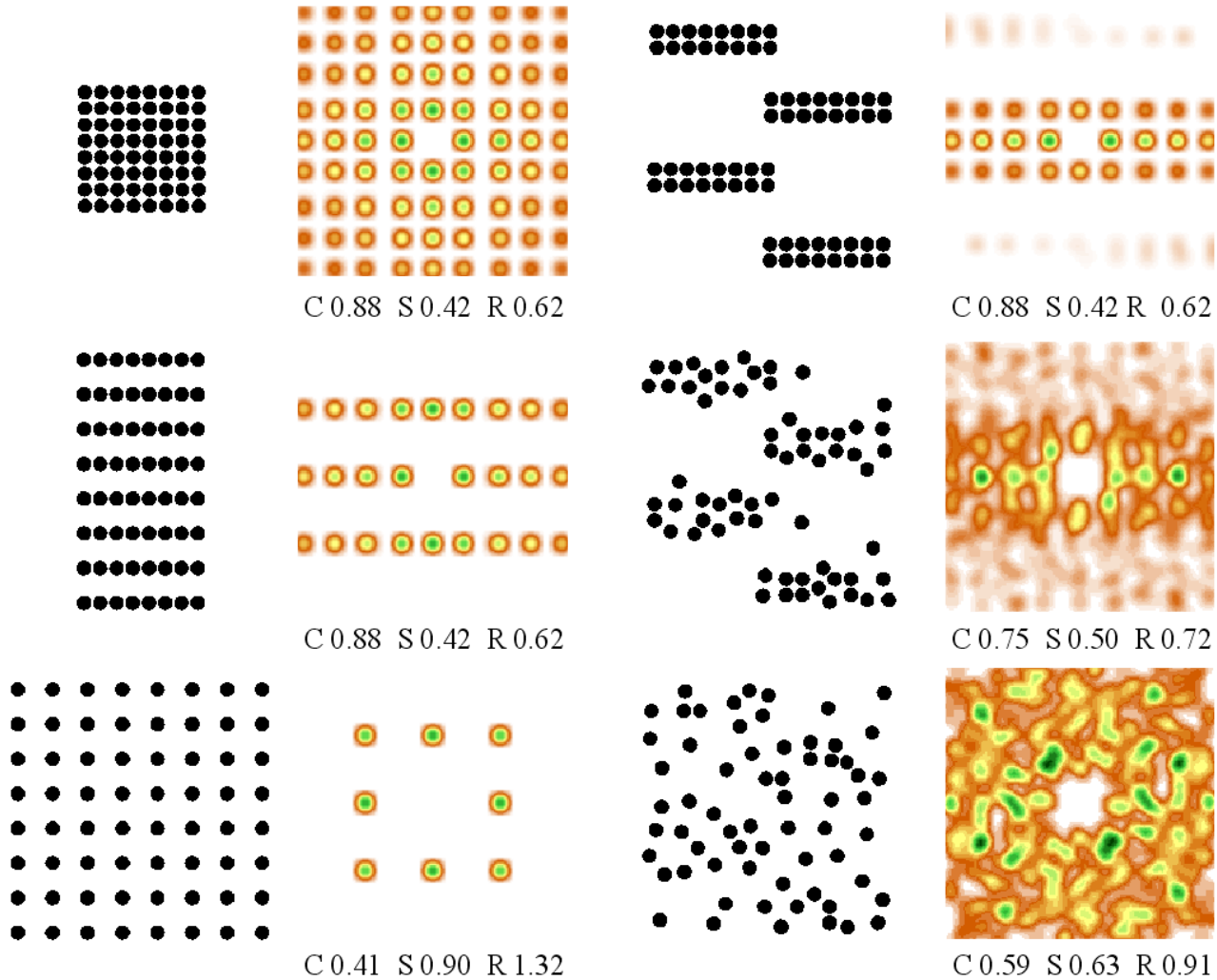
$R = 1.630 @ 114,3^\circ$

$R = 1.645 @ 114,1^\circ$

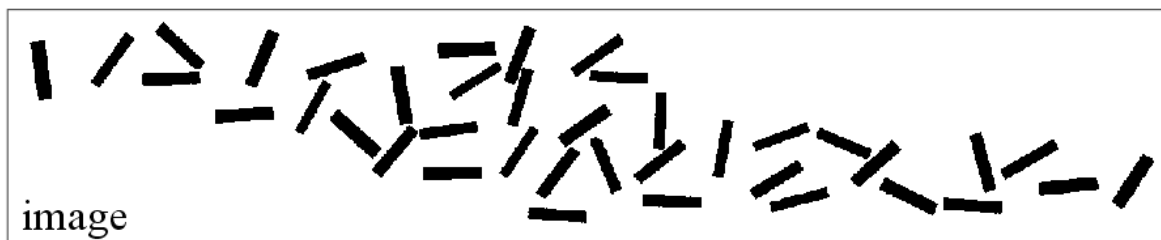


**Inertia
tensor**

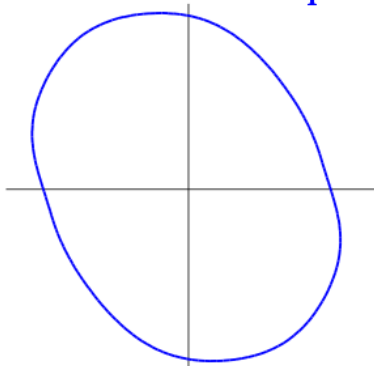
Spatial distribution and Compaction



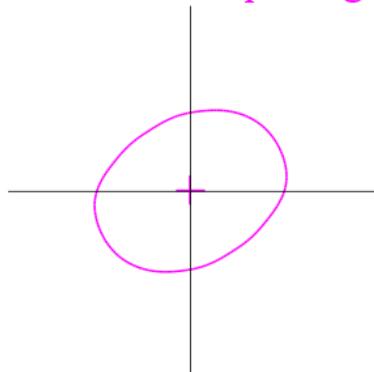
Autocorrelation / intercepts



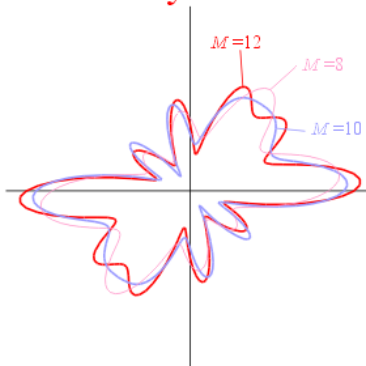
count of intercept



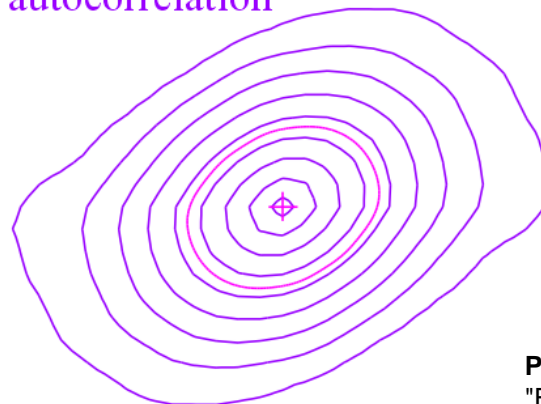
mean intercept length



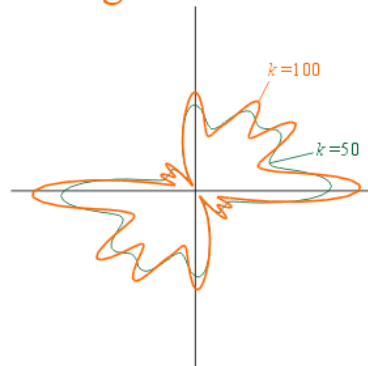
boundary direction



autocorrelation

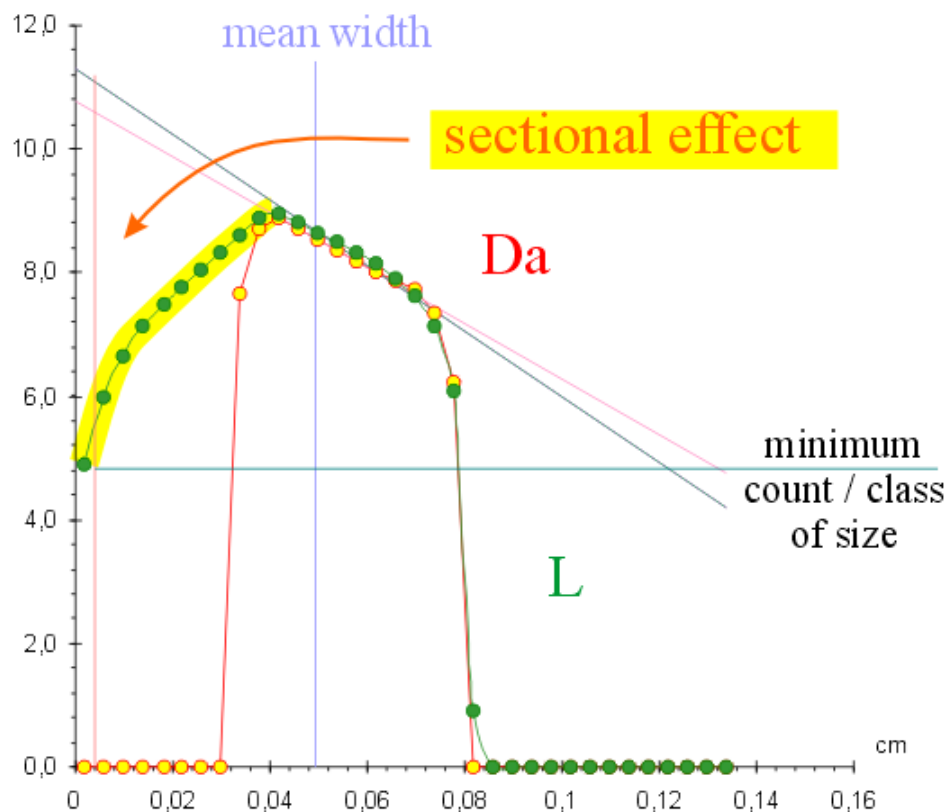


long axis direction

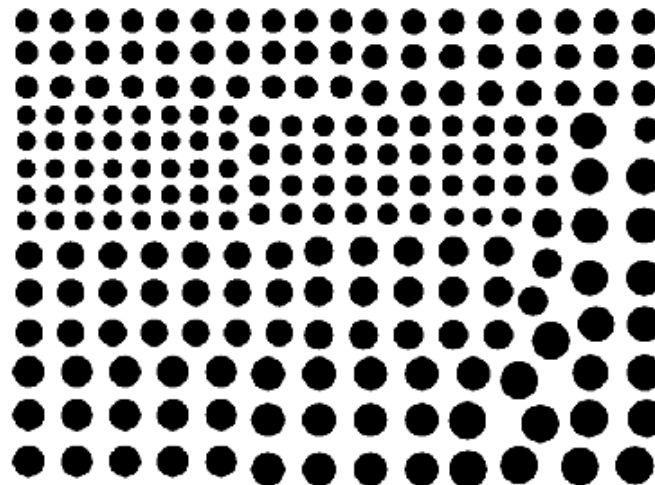


CSD

$\ln(n)$ with n =number of object / surface area / class length

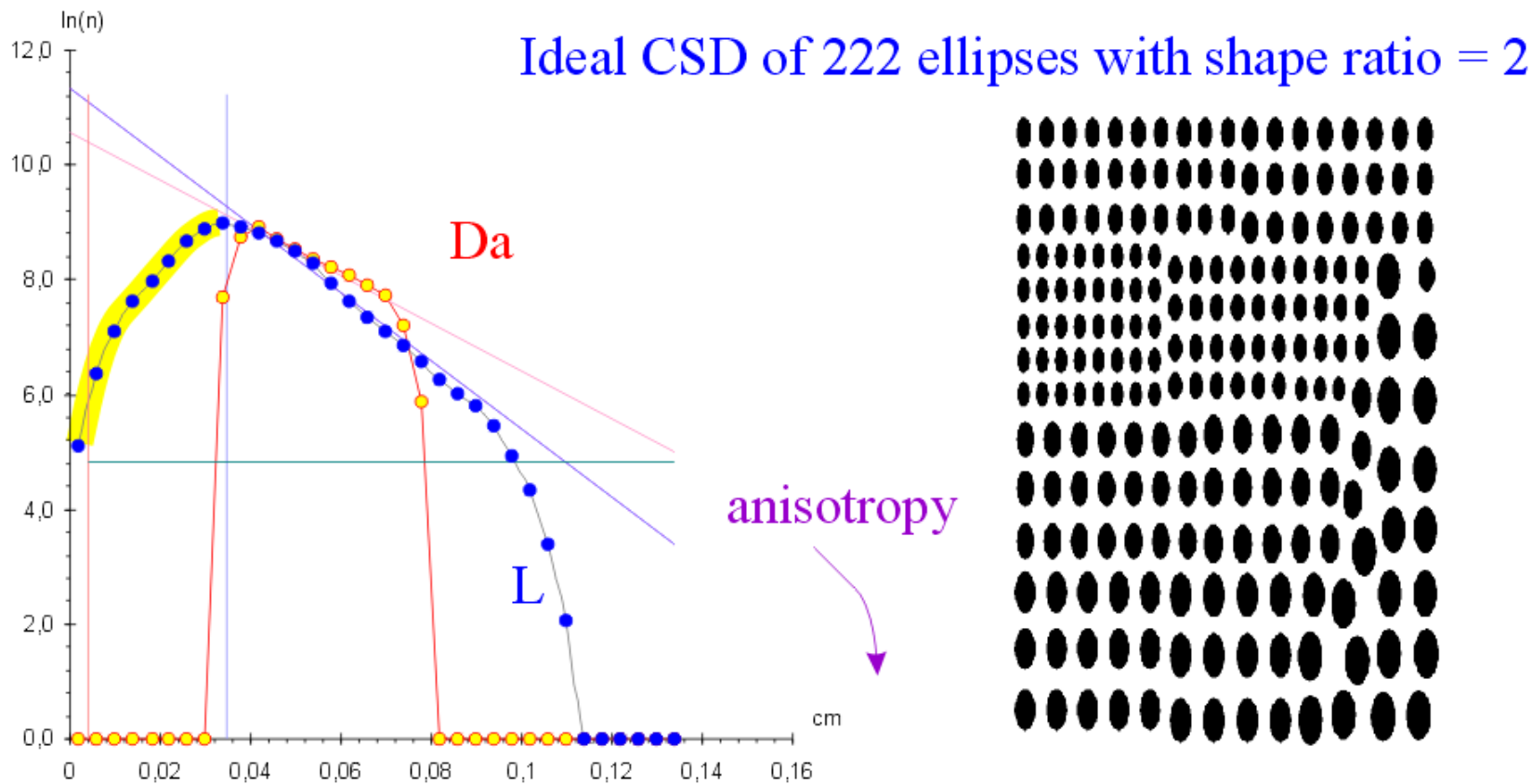


Ideal CSD of 222 circles



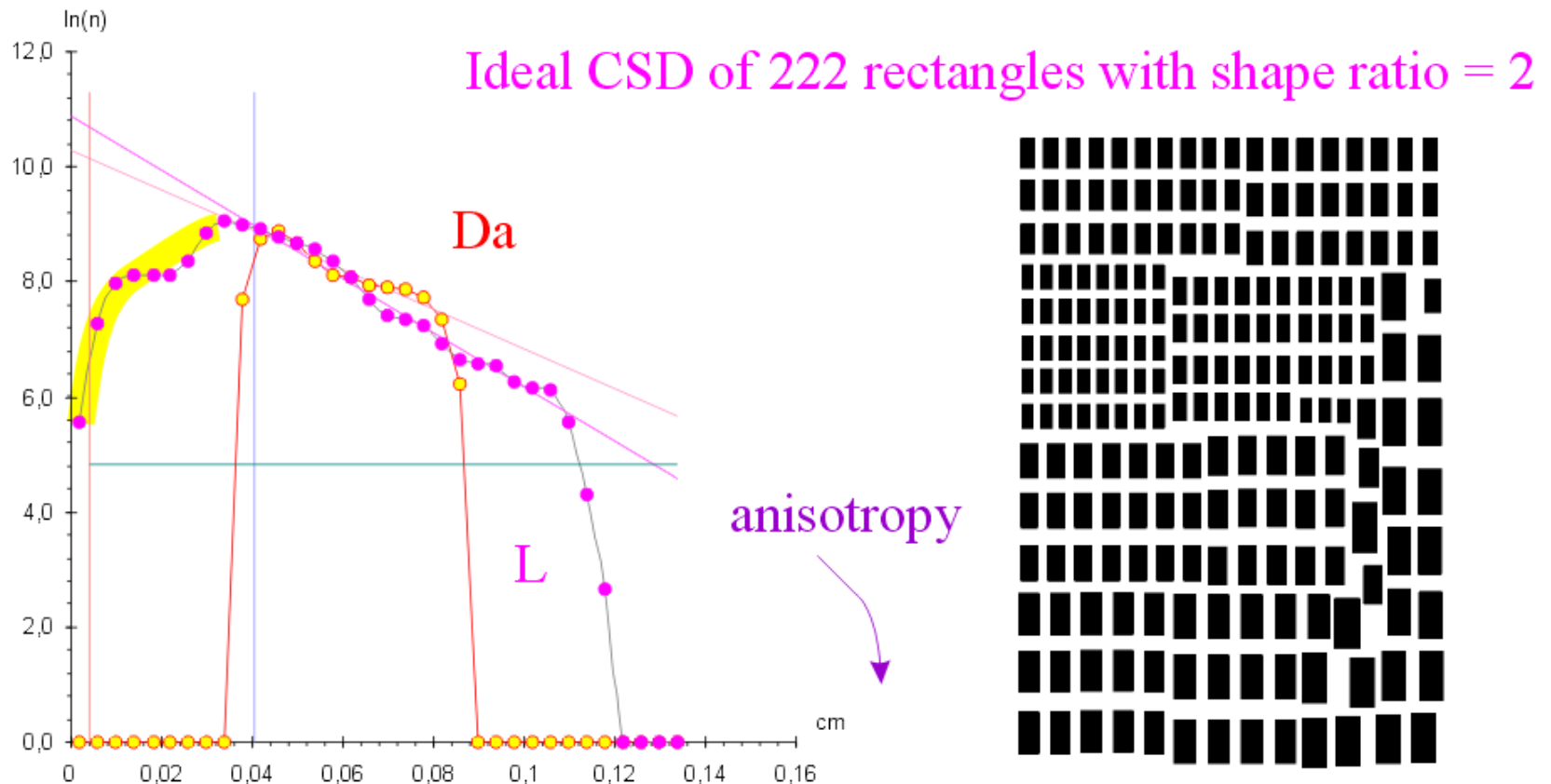
CSD

$\ln(n)$ with n =number of object / surface area / class length



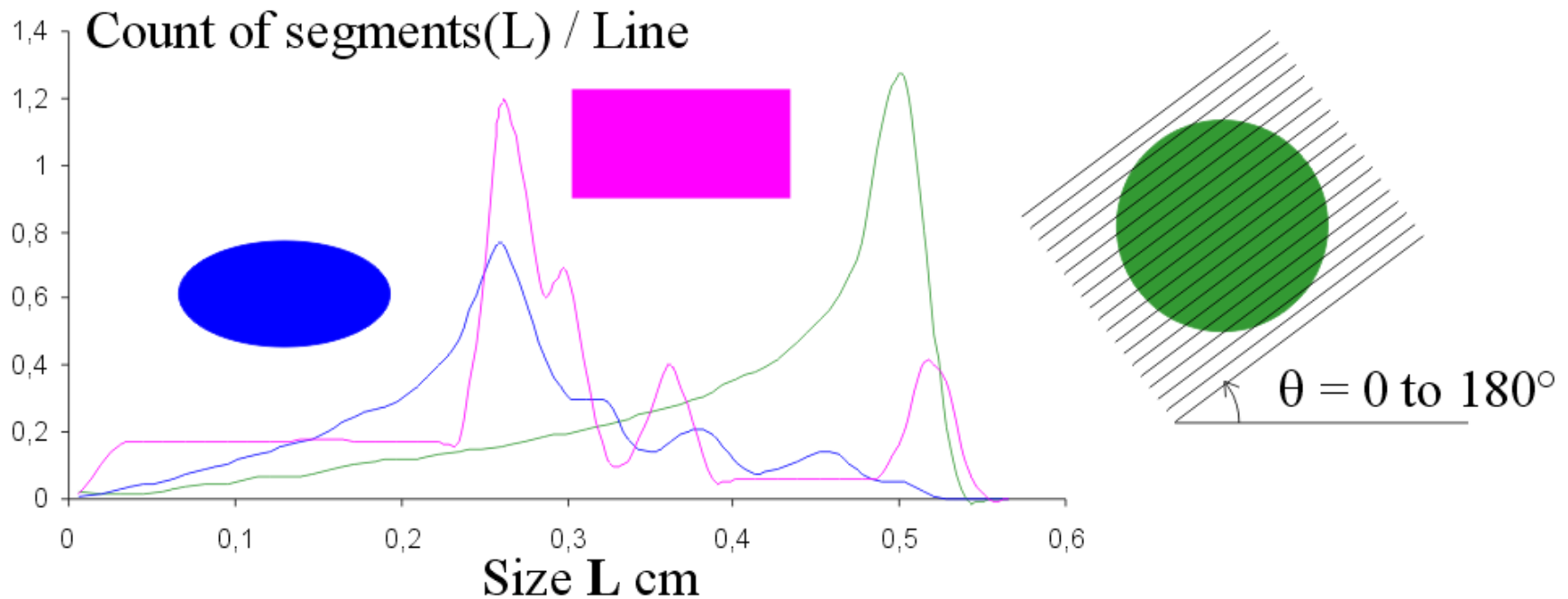
CSD

$\ln(n)$ with n =number of object / surface area / class length



CSD

Histogram of segments cut by a set of lines between 0 and 180°



Patrick Launeau

Quantitative Image Analysis of Minerals and Rocks

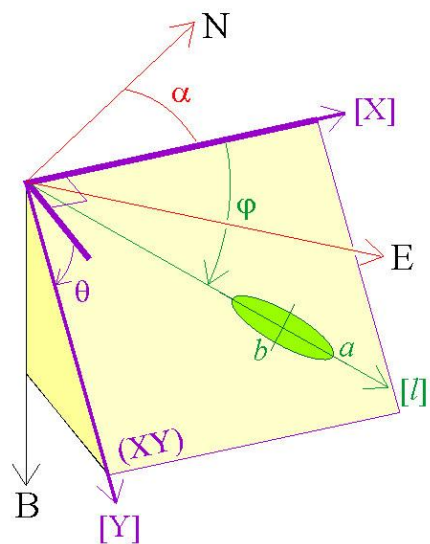
Fabric analysis

- Shape preferred orientation (SPO) vs. strain quantification.
- Intercepts in digital images : a tool to analyze interconnection of grains in rocks vs. inertia tensor of individualized grains
- SPO vs Spatial distribution (Fry)
- Ellipsoid of SPO and strain by combining 3 \perp images.



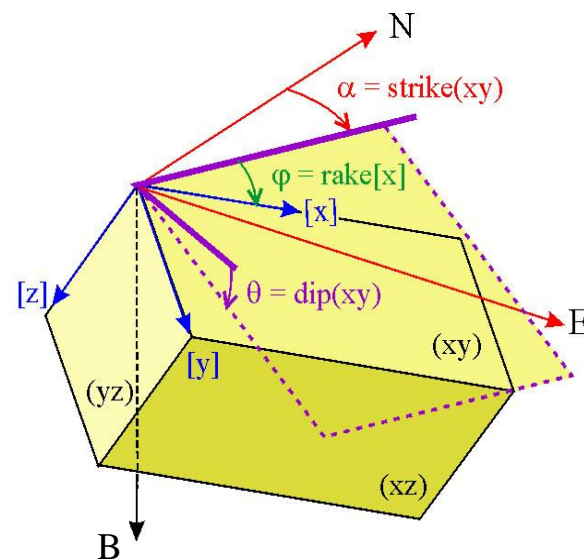
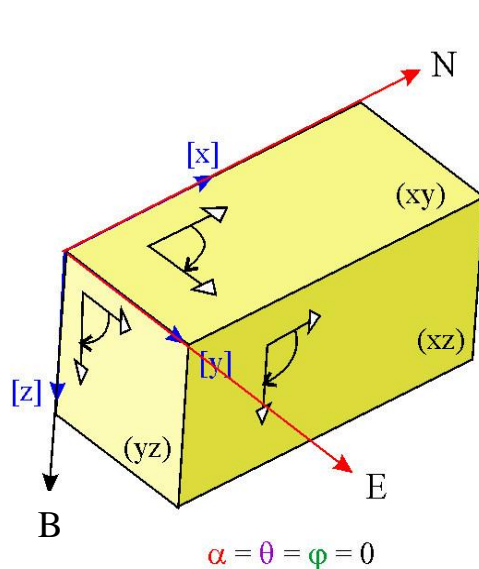
From 2D image analysis to 3D ellipsoid construction

$$\mathbf{R}_L = \begin{bmatrix} \cos\alpha\cos\varphi - \sin\alpha\cos\theta\sin\varphi \\ \sin\alpha\cos\varphi + \cos\alpha\cos\theta\sin\varphi \\ \sin\theta\sin\varphi \end{bmatrix}$$



North, East, Down convention

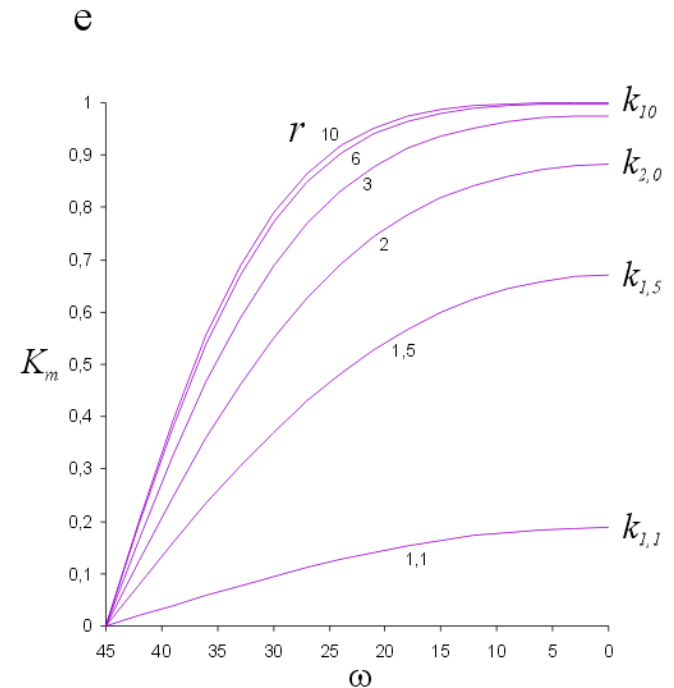
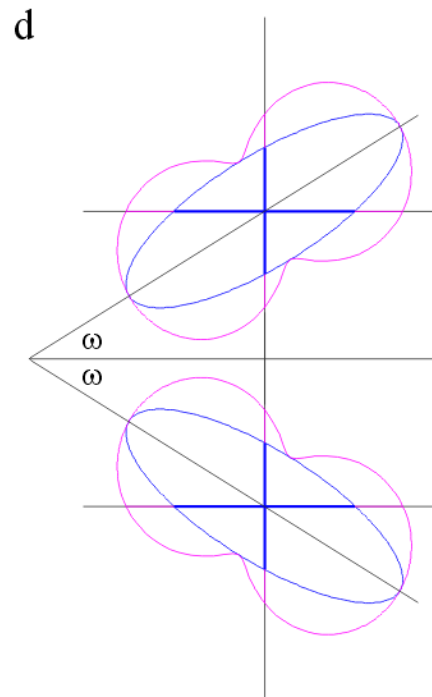
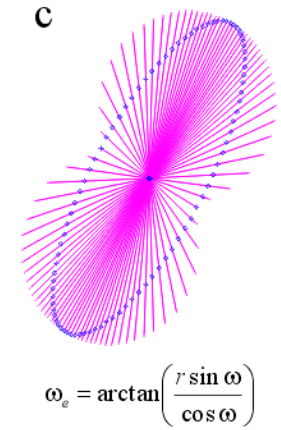
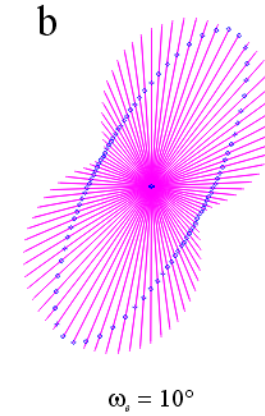
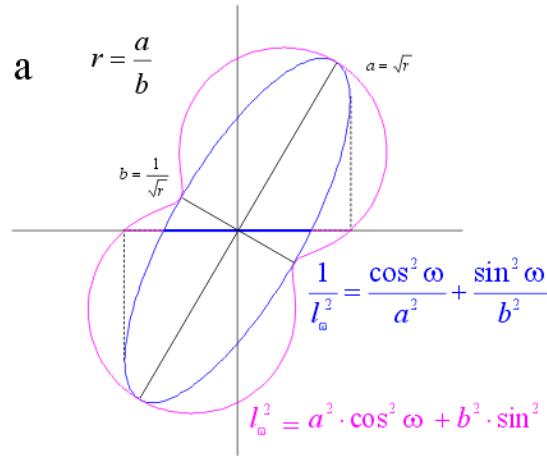
Euler angles : strike α , dip θ , pitch or rake φ



P. Launeau, P.-Y. F. Robin
(2005) "Determination of fabric and strain ellipsoids from measured sectional ellipses—implementation and applications". *Journal of Structural Geology*, 27, 2223-2233

$$\mathbf{R}_V = \begin{bmatrix} \cos\alpha\cos\varphi - \sin\alpha\cos\theta\sin\varphi & -\cos\alpha\sin\varphi - \sin\alpha\cos\theta\cos\varphi & \sin\alpha\sin\theta \\ \sin\alpha\cos\varphi + \cos\alpha\cos\theta\sin\varphi & -\sin\alpha\sin\varphi + \cos\alpha\cos\theta\cos\varphi & -\cos\alpha\sin\theta \\ \sin\theta\sin\varphi & \sin\theta\cos\varphi & \cos\theta \end{bmatrix}$$

From 2D ellipse to 3D ellipsoid

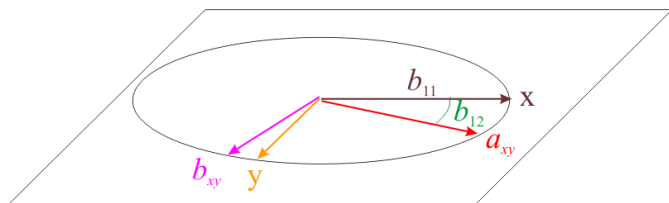


P. Launeau (2004) "Mise en évidence des écoulements magmatiques par analyse d'images 2-D des distributions 3-D d'Orientations Préférentielles de Formes". *Bull. Soc. Géol. Fr.*, 175, 331-350

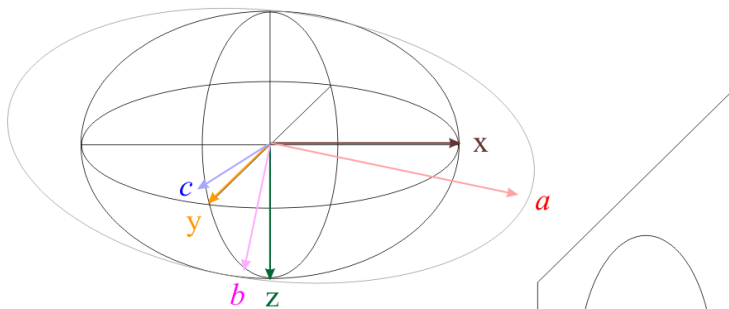
$$\frac{1}{l(\omega)^2} = \frac{\cos^2 \omega}{a^2} + \frac{\sin^2 \omega}{b^2} \quad \text{2D ellipses} \quad \& \quad \text{3D ellipsoids} \quad \frac{1}{l(x,y,z)^2} = \frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2}$$

$$\mathbf{B}_{12} = \begin{bmatrix} b_{11} & b_{12} \\ b_{12} & b_{22} \end{bmatrix} = \begin{bmatrix} \cos \varphi & \sin \varphi \\ -\sin \varphi & \cos \varphi \end{bmatrix} \cdot \begin{bmatrix} 1/a^2 & 0 \\ 0 & 1/b^2 \end{bmatrix} \cdot \begin{bmatrix} \cos \varphi & -\sin \varphi \\ \sin \varphi & \cos \varphi \end{bmatrix}$$

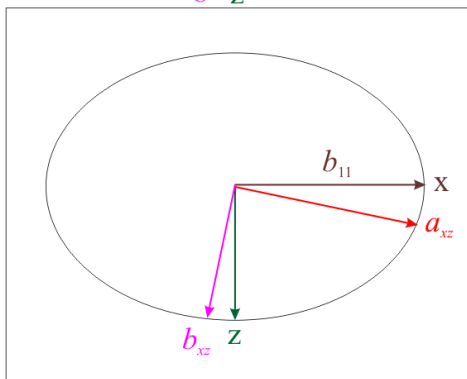
$$\mathbf{B}_{123} = \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{12} & b_{22} & b_{23} \\ b_{13} & b_{23} & b_{33} \end{bmatrix} = \mathbf{R}_V^T \cdot \begin{bmatrix} 1/a^2 & 0 & 0 \\ 0 & 1/b^2 & 0 \\ 0 & 0 & 1/c^2 \end{bmatrix} \cdot \mathbf{R}_V$$



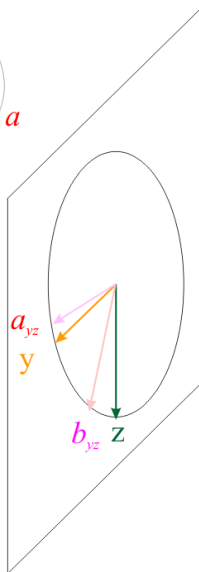
$$\mathbf{B}_{12} = \begin{bmatrix} \bar{b}_{11} & \bar{b}_{12} \\ b_{12} & b_{22} \end{bmatrix} = \begin{bmatrix} \cos \varphi_{12} & \sin \varphi_{12} \\ -\sin \varphi_{12} & \cos \varphi_{12} \end{bmatrix} \cdot \begin{bmatrix} 1/a_{12}^2 & 0 \\ 0 & 1/b_{12}^2 \end{bmatrix} \cdot \begin{bmatrix} \cos \varphi_{12} & -\sin \varphi_{12} \\ \sin \varphi_{12} & \cos \varphi_{12} \end{bmatrix}$$



$$\mathbf{B}_{13} = \begin{bmatrix} \bar{b}_{11} & b_{13} \\ b_{13} & b_{33} \end{bmatrix} = \begin{bmatrix} \cos \varphi_{13} & \sin \varphi_{13} \\ -\sin \varphi_{13} & \cos \varphi_{13} \end{bmatrix} \cdot \begin{bmatrix} 1/a_{13}^2 & 0 \\ 0 & 1/b_{13}^2 \end{bmatrix} \cdot \begin{bmatrix} \cos \varphi_{13} & -\sin \varphi_{13} \\ \sin \varphi_{13} & \cos \varphi_{13} \end{bmatrix}$$



$$\mathbf{B}_{23} = \begin{bmatrix} b_{22} & b_{23} \\ b_{23} & b_{33} \end{bmatrix} = \begin{bmatrix} \cos \varphi_{23} & \sin \varphi_{23} \\ -\sin \varphi_{23} & \cos \varphi_{23} \end{bmatrix} \cdot \begin{bmatrix} 1/a_{23}^2 & 0 \\ 0 & 1/b_{23}^2 \end{bmatrix} \cdot \begin{bmatrix} \cos \varphi_{23} & -\sin \varphi_{23} \\ \sin \varphi_{23} & \cos \varphi_{23} \end{bmatrix}$$



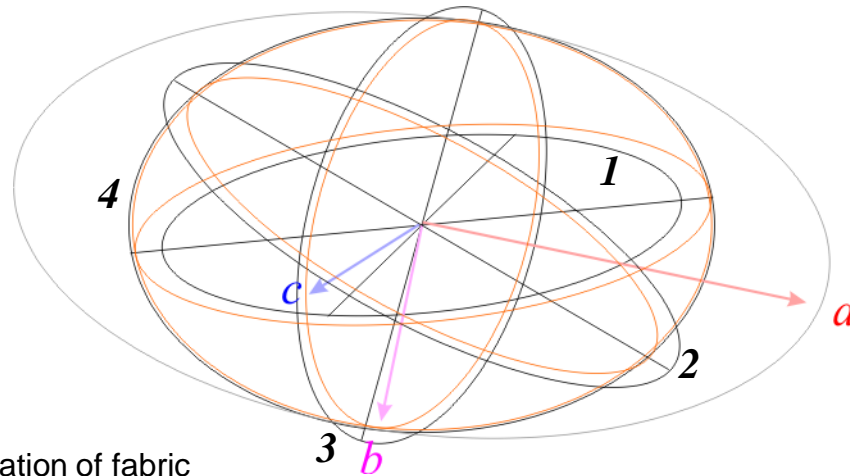
$$\mathbf{B} = \begin{bmatrix} \bar{b}_{11} & \bar{b}_{12} & b_{13} \\ b_{12} & \bar{b}_{22} & b_{23} \\ b_{13} & b_{23} & \bar{b}_{33} \end{bmatrix}$$

(x,y,z) (1,2,3)

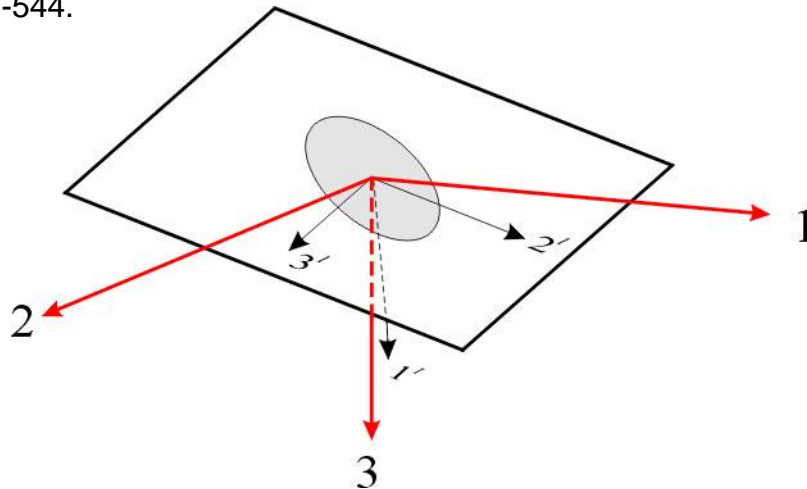
Launeau, P. and Cruden, A.R. (1998). – Magmatic fabric acquisition mechanism in a syenite: Results of a combined anisotropy of magnetic susceptibility and image analysis study. J. Geophys. Res. 103, 5067-5089.

2D ellipses & 3D ellipsoids

$I = 1 \text{ to } 4$



Robin, P.-Y.F. (2002). – Determination of fabric and strain ellipsoids from measured sectional ellipses – Theory. *Journal of Structural Geology*, 24, 531-544.



$$\mathbf{B}_{23}^I = \mathbf{L}_{23}^{I\top} \cdot \mathbf{B}_{123} \cdot \mathbf{L}_{23}^I + \mathbf{X}^I$$

with

$$\mathbf{X}^I = \begin{bmatrix} \chi_{22}^I & \chi_{23}^I \\ \chi_{23}^I & \chi_{33}^I \end{bmatrix}$$

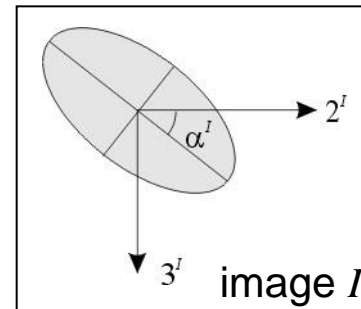
deviations to
measurements

$$\mathbf{B}_{23}^I = \begin{bmatrix} b_{22}^I & b_{23}^I \\ b_{23}^I & b_{33}^I \end{bmatrix}$$

of the section

$$\mathbf{L}_{23}^I = \begin{bmatrix} l_{21}^I & l_{31}^I \\ l_{22}^I & l_{32}^I \\ l_{23}^I & l_{33}^I \end{bmatrix}$$

of ellipsoid \mathbf{B}_{123}



Scale factors of the Robin 2002 method

$$\mathbf{B}_{23}^I = \begin{bmatrix} \cos\varphi & \sin\varphi \\ -\sin\varphi & \cos\varphi \end{bmatrix} \cdot \begin{bmatrix} 1/a^2 & 0 \\ 0 & 1/b^2 \end{bmatrix} \cdot \begin{bmatrix} \cos\varphi & -\sin\varphi \\ \sin\varphi & \cos\varphi \end{bmatrix}$$

$$\frac{1}{\bar{a}^I} \cdot \begin{bmatrix} 1/\rho & 0 \\ 0 & \rho \end{bmatrix}$$

$$\bar{n}^I \cdot \begin{bmatrix} 1/\rho & 0 \\ 0 & \rho \end{bmatrix}$$

Scale factor available

length a et b

mean surface area $\bar{a}^I = A^I / j^I$

with $\rho = \frac{a}{b}$ the shape ratio

mean density $\bar{n}^I = j^I / A^I$

Robin, P.-Y.F. (2002). –
Determination of fabric and strain ellipsoids from measured sectional ellipses – Theory. *Journal of Structural Geology*, 24, 531-544.

$$\mathbf{B}_{23}^I = f^I \cdot \begin{bmatrix} \cos\varphi & \sin\varphi \\ -\sin\varphi & \cos\varphi \end{bmatrix} \cdot \begin{bmatrix} 1/\rho & 0 \\ 0 & \rho \end{bmatrix} \cdot \begin{bmatrix} \cos\varphi & -\sin\varphi \\ \sin\varphi & \cos\varphi \end{bmatrix}$$

No scale factor available

$\sqrt{F_{\min}^I}$ Compatibility index of section I

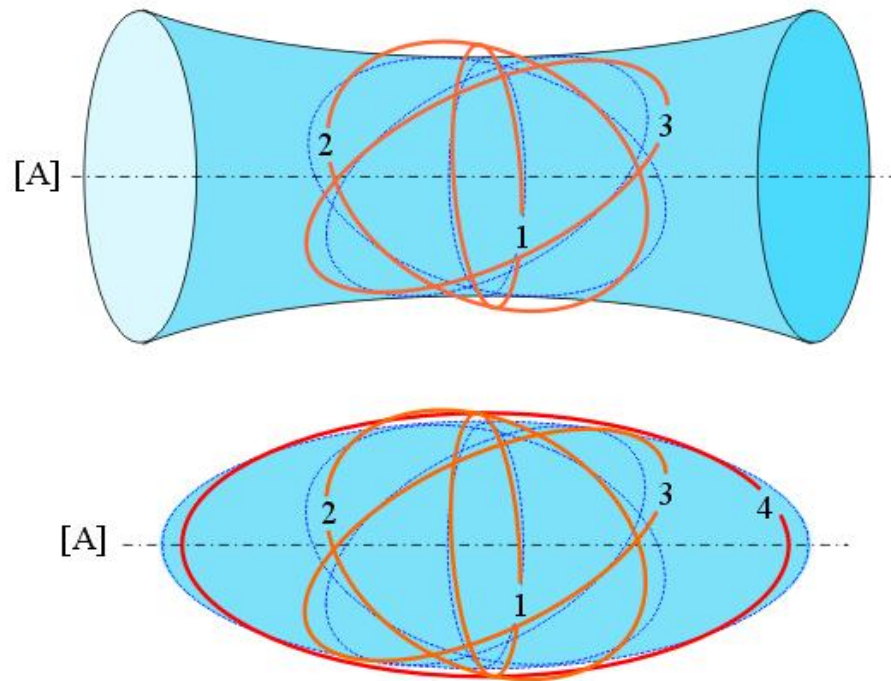
In case of 3 perpendicular sections

$$\begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{23} & b_{33} \end{bmatrix} = \begin{bmatrix} (b_{33}^2 + b_{22}^3)/2 & b_{23}^3 & b_{23}^2 \\ b_{23}^3 & (b_{22}^1 + b_{33}^3)/2 & b_{23}^1 \\ b_{23}^2 & b_{23}^1 & (b_{33}^1 + b_{22}^2)/2 \end{bmatrix}$$

Compatibility index of all sections

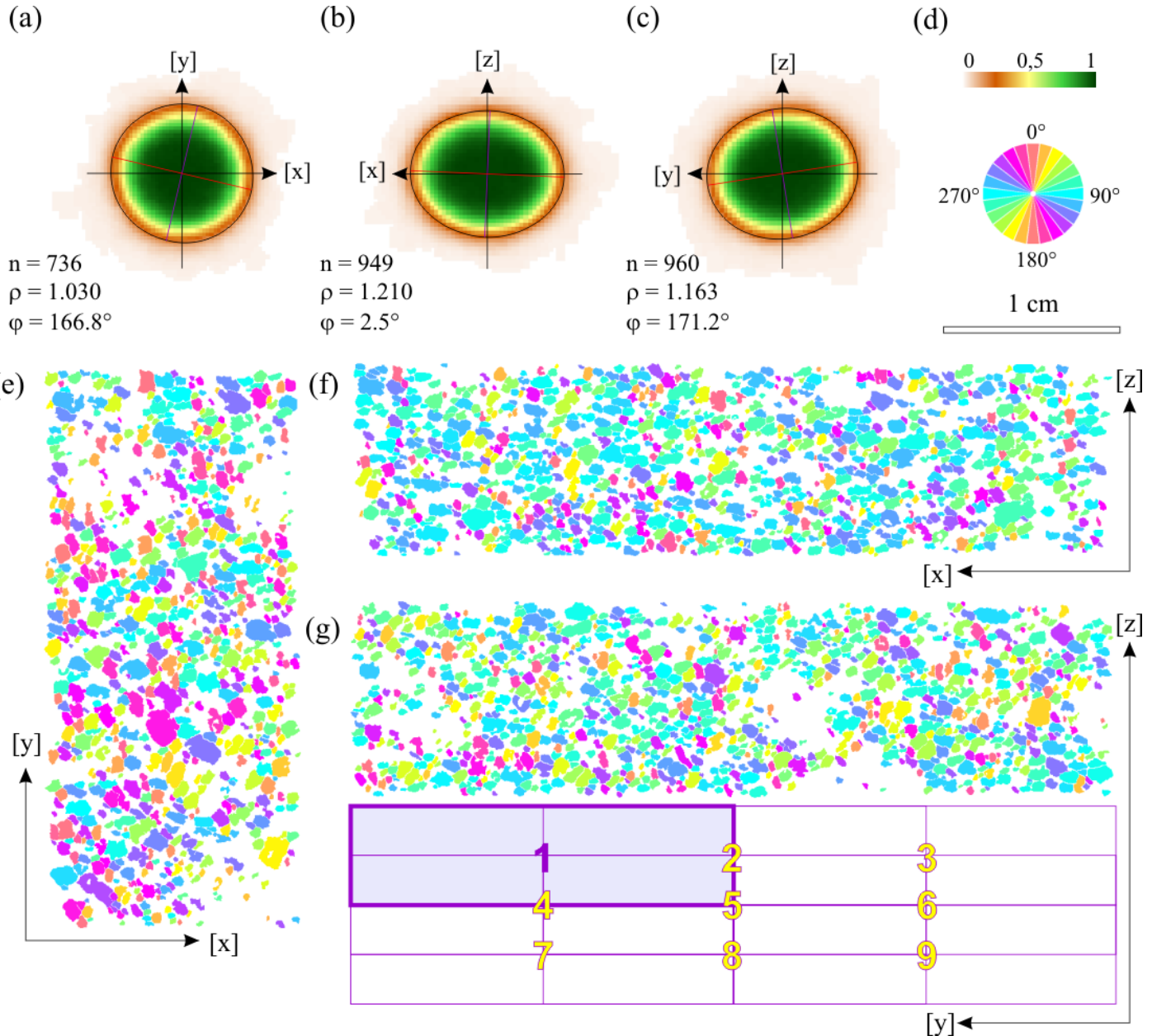
$$\tilde{F} = \frac{1}{6} \frac{(b_{33}^2 - b_{22}^3)^2 + (b_{22}^1 - b_{33}^3)^2 + (b_{33}^1 - b_{22}^2)^2}{(b_1)^2 + (b_2)^2 + (b_3)^2}$$

Convergence of the Robin 2002 method



Robin, P.-Y.F. (2002). – Determination of fabric and strain ellipsoids from measured sectional ellipses – Theory. *Journal of Structural Geology*, 24, 531-544.

Application to gabbro-norites of the Bushveld complex



Orthopyroxene

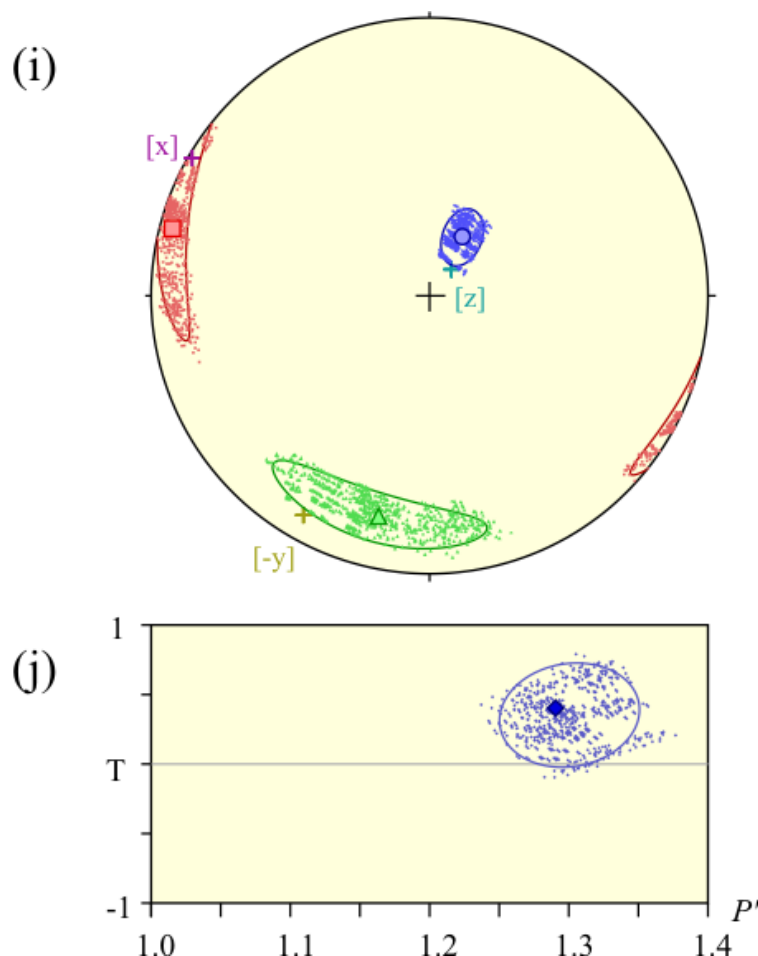
P. Launeau, P.-Y. F. Robin
(2005) "Determination of fabric and strain ellipsoids from measured sectional ellipses—implementation and applications". *Journal of Structural Geology*, 27, 2223-2233

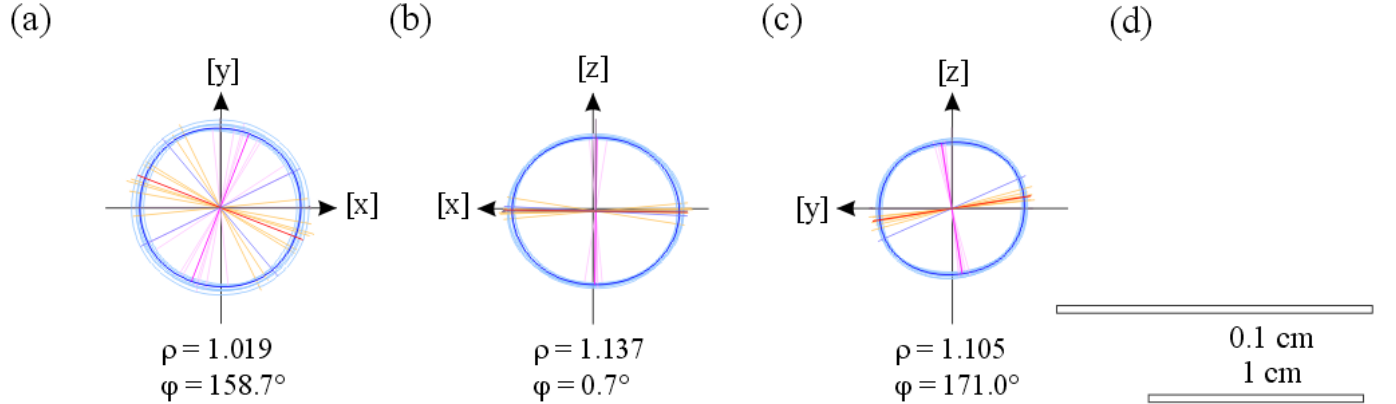
Application to gabbronorites of the Bushveld complex

(h)

	A	B	C	$\sqrt{\tilde{F}}$
L. norm	1.113	1.034	0.869	2.5%
trend	284.7	192.9	29.0	
plunge	5.1	19.1	70.2	
A/C	1.282		Flinn	0.405
A/B	1.077		P'	1.291
B/C	1.192		T	0.396

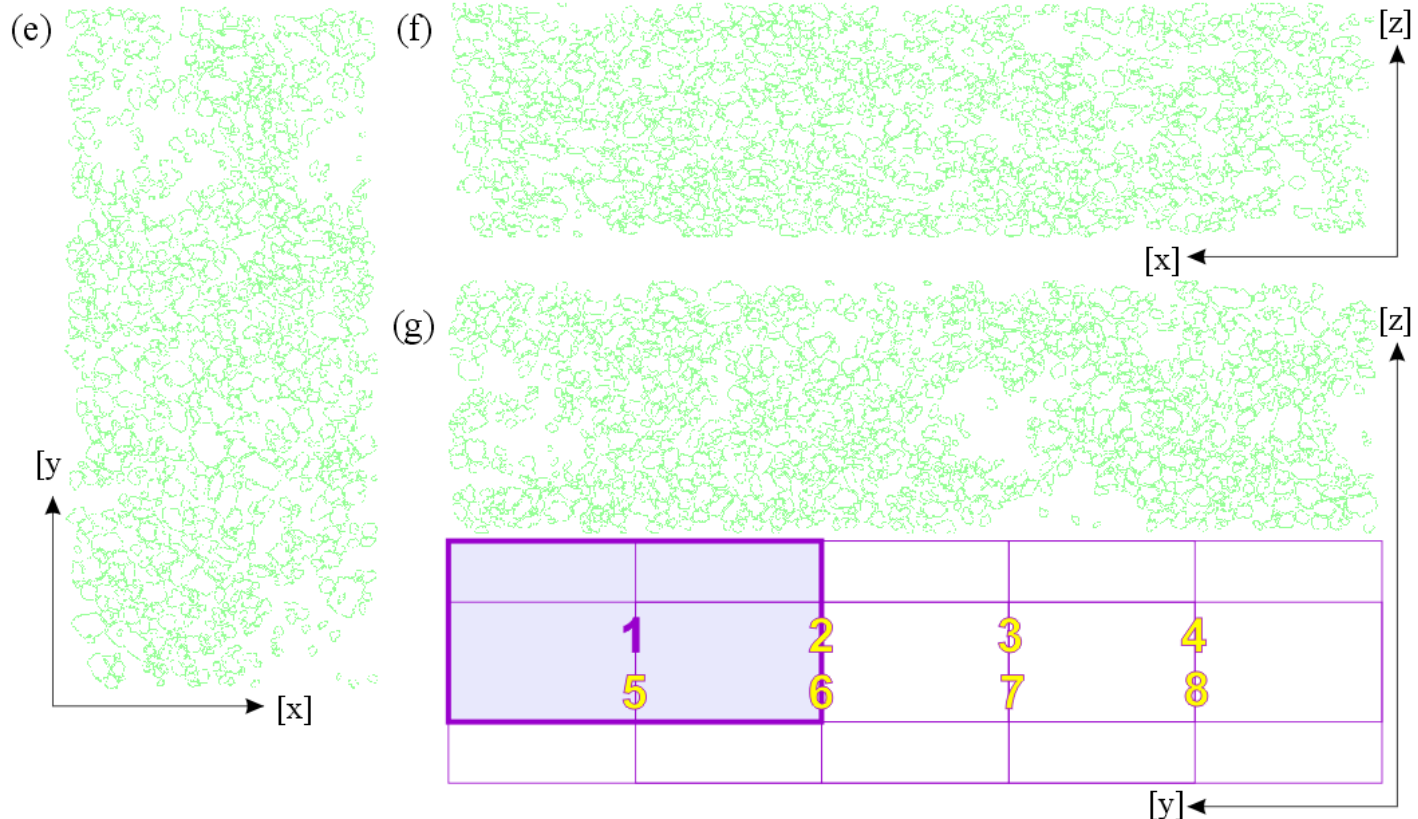
	A	B	C	729 combinations	
L. norm	1.120	1.030	0.867	\bar{X}	
	0.028	0.033	0.021	2σ	
trend	285.9	194.2	29.1	\bar{X}	
plunge	4.7	19.0	70.4	\bar{X}	
	27.4	27.4	8.8	$2\sigma_1$	
	5.6	8.8	5.7	$2\sigma_2$	
	\bar{X}	2σ		\bar{X}	2σ
A/C	1.292	0.048	$\sqrt{\tilde{F}}$	2.6%	3.5%
A/B	1.087	0.057	P'	1.301	0.050
B/C	1.189	0.060	T	0.352	0.375





Application to gabbro-norites of the Bushveld complex

Orthopyroxene



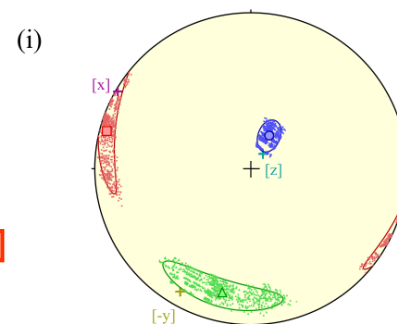
P. Launeau, P.-Y. F. Robin
(2005) "Determination of fabric and strain ellipsoids from measured sectional ellipses—implementation and applications". *Journal of Structural Geology*, 27, 2223-2233

Application to gabbronorites of the Bushveld complex

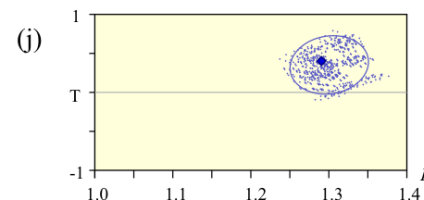
(h)

	A	B	C	\sqrt{F}
L. norm	1.113	1.034	0.869	2.5%
trend	284.7	192.9	29.0	
plunge	5.1	19.1	70.2	
A/C	1.282		Flinn	0.405
A/B	1.077		P'	1.291
B/C	1.192		T	0.396

	A	B	C	729 combinations	
L. norm	1.120	1.030	0.867	\bar{X}	
	0.028	0.033	0.021	2σ	
trend	285.9	194.2	29.1	\bar{X}	
plunge	4.7	19.0	70.4	\bar{X}	
	27.4	27.4	8.8	$2\sigma_1$	
	5.6	8.8	5.7	$2\sigma_2$	
	\bar{X}	2σ	\bar{X}	2σ	
A/C	1.292	0.048	\sqrt{F}	2.6%	3.5%
A/B	1.087	0.057	P'	1.301	0.050
B/C	1.189	0.060	T	0.352	0.375



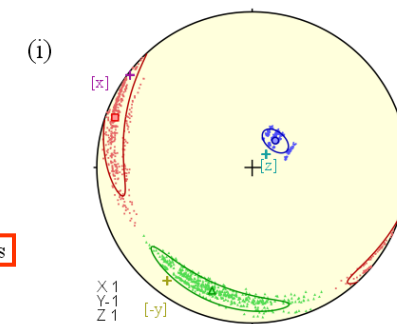
Inertia
tensors



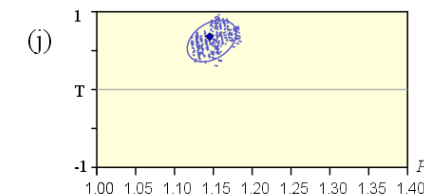
(h)

	A	B	C	\sqrt{F}
L. norm	1.050	1.029	0.926	1.4%
trend	290.0	197.9	40.8	
plunge	6.8	17.2	71.5	
A/C	1.134		Flinn	0.179
A/B	1.020		P'	1.145
B/C	1.112		T	0.680

	A	B	C	512 combinations	
L. norm	1.052	1.027	0.925	\bar{X}	
	0.011	0.014	0.015	2σ	
trend	289.6	197.6	41.3	\bar{X}	
plunge	6.9	16.8	71.5	\bar{X}	
	33.2	33.0	7.8	$2\sigma_1$	
	6.8	5.9	4.9	$2\sigma_2$	
	\bar{X}	2σ	\bar{X}	2σ	
A/C	1.138	0.027	\sqrt{F}	1.7%	2.1%
A/B	1.024	0.018	P'	1.148	0.031
B/C	1.111	0.031	T	0.625	0.268



Intercepts



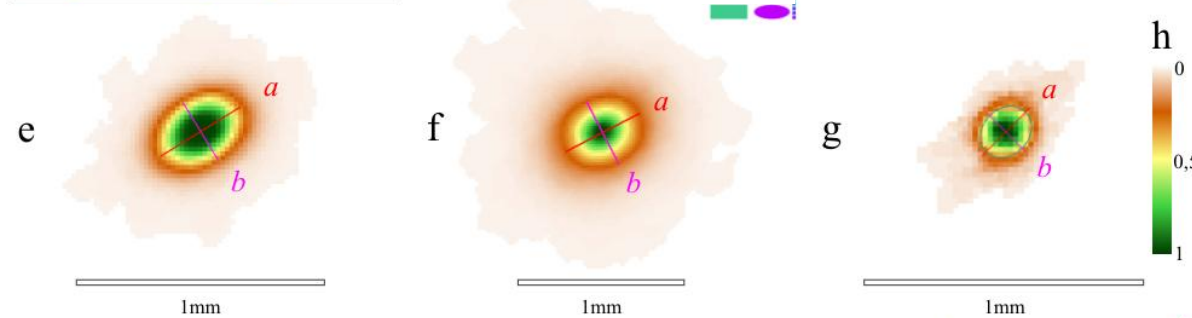
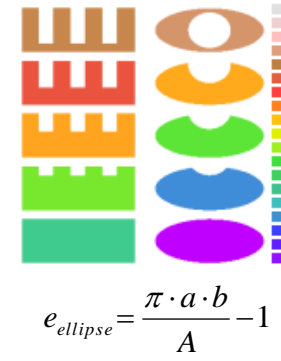
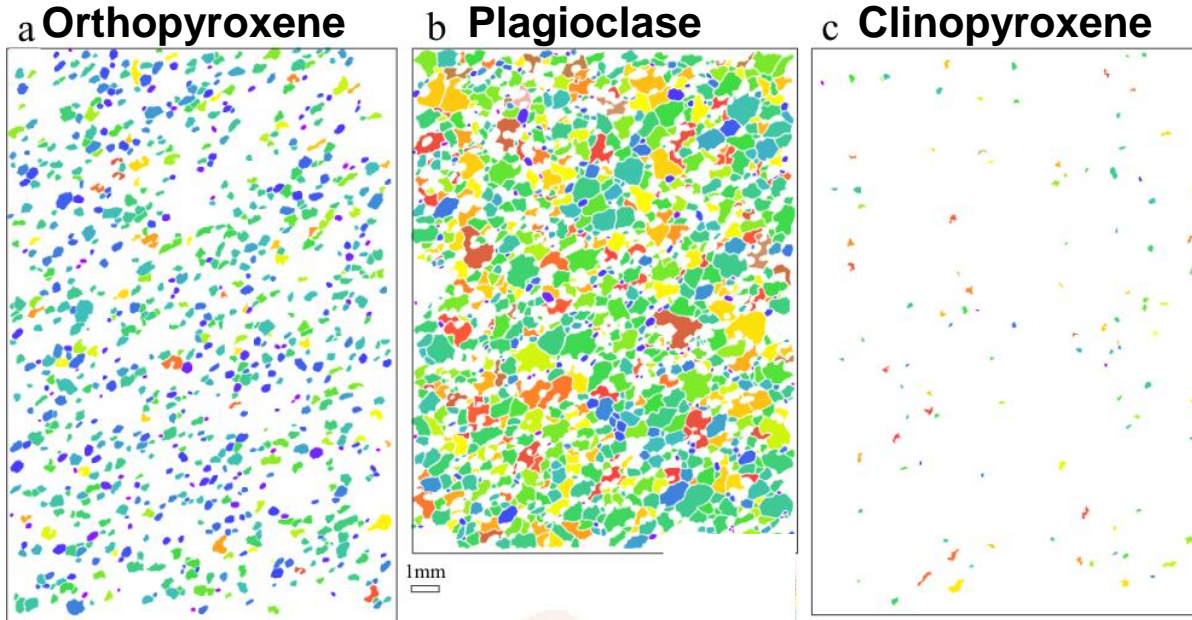
P. Launeau, P.-Y. F. Robin
(2005) "Determination of
fabric and strain ellipsoids
from measured sectional
ellipses—implementation
and applications". *Journal of
Structural Geology*, 27,
2223-2233

Application to gabbro-norites of the Bushveld complex

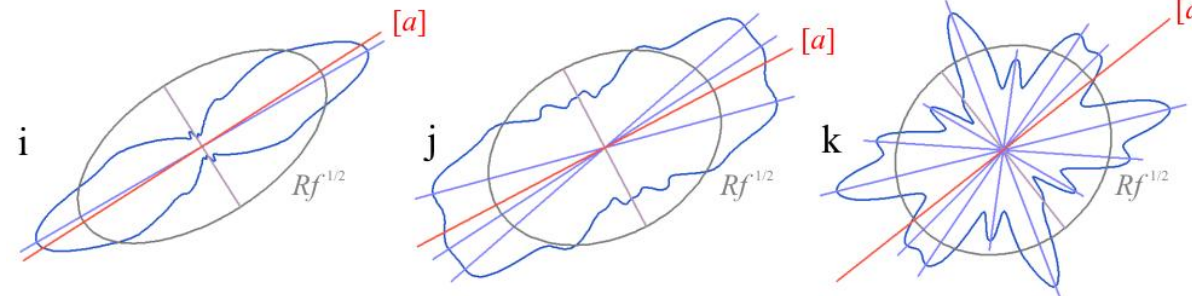
P. Launeau (2004) "Mise en évidence des écoulements magmatiques par analyse d'images 2-D des distributions 3-D d'orientations Préférentielles de Formes". *Bull. Soc. Géol. Fr.*, 175, 331-350

SPO

PO



pixel density of crystal stack

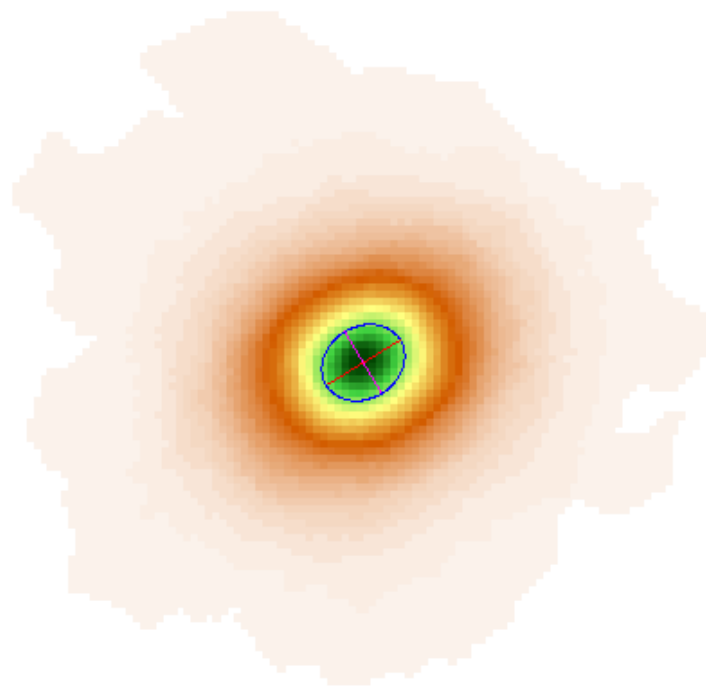
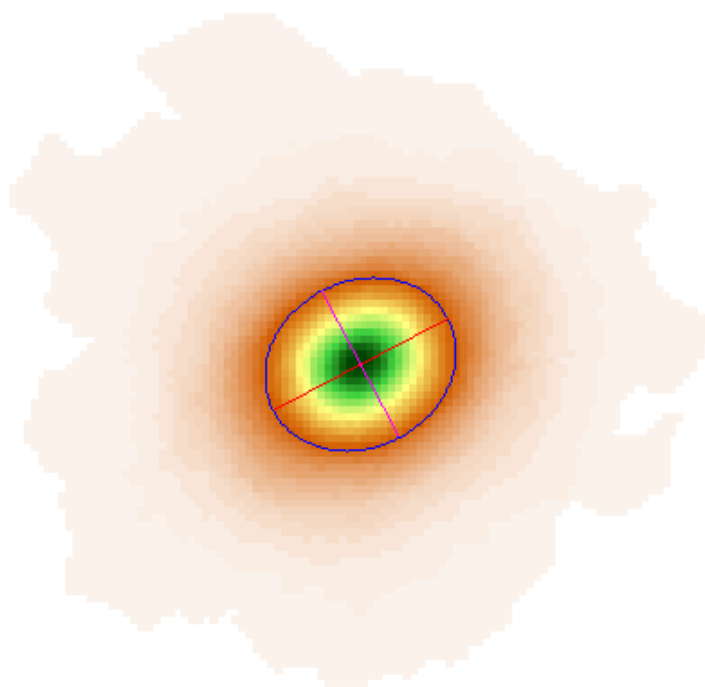
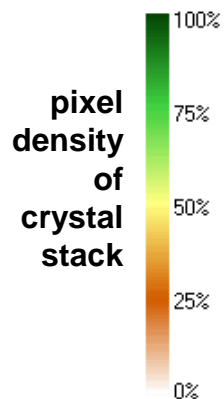


Application to gabbro-norites of the Bushveld complex

Plagioclase

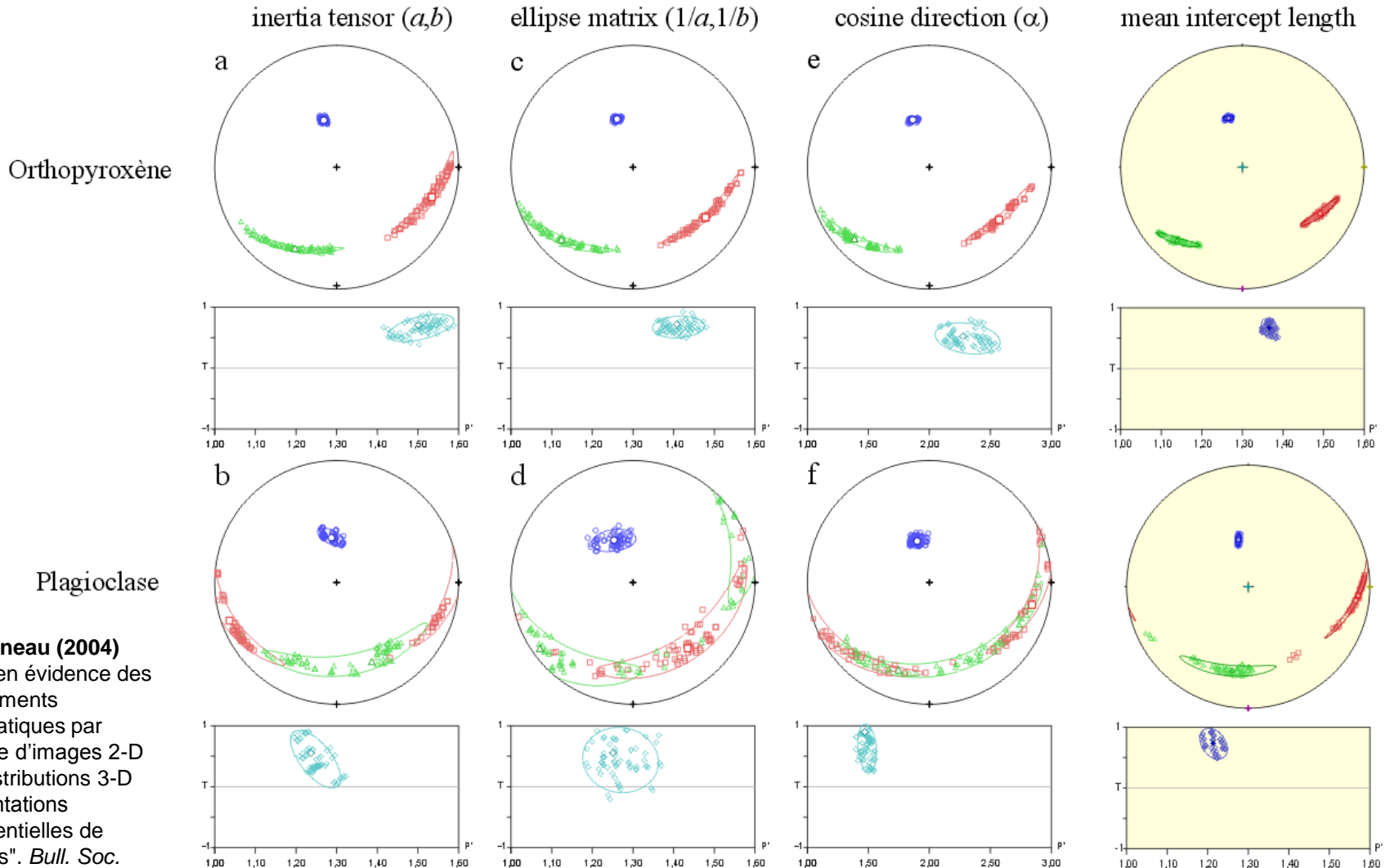
$$\mathbf{M}_T = \frac{1}{4 \cdot A} \begin{bmatrix} a^2 \cdot \cos^2 \varphi + b^2 \cdot \sin^2 \varphi & (a^2 - b^2) \cdot \cos \varphi \cdot \sin \varphi \\ (a^2 - b^2) \cdot \cos \varphi \cdot \sin \varphi & a^2 \cdot \sin^2 \varphi + b^2 \cdot \cos^2 \varphi \end{bmatrix}$$

$$\mathbf{M}_E = A \cdot \begin{bmatrix} \frac{1}{a^2} \cdot \cos^2 \varphi + \frac{1}{b^2} \cdot \sin^2 \varphi & \left(\frac{1}{a^2} - \frac{1}{b^2} \right) \cdot \cos \varphi \cdot \sin \varphi \\ \left(\frac{1}{a^2} - \frac{1}{b^2} \right) \cdot \cos \varphi \cdot \sin \varphi & \frac{1}{a^2} \cdot \sin^2 \varphi + \frac{1}{b^2} \cdot \cos^2 \varphi \end{bmatrix}$$



0,1 cm

Application to gabbronorites of the Bushveld complex

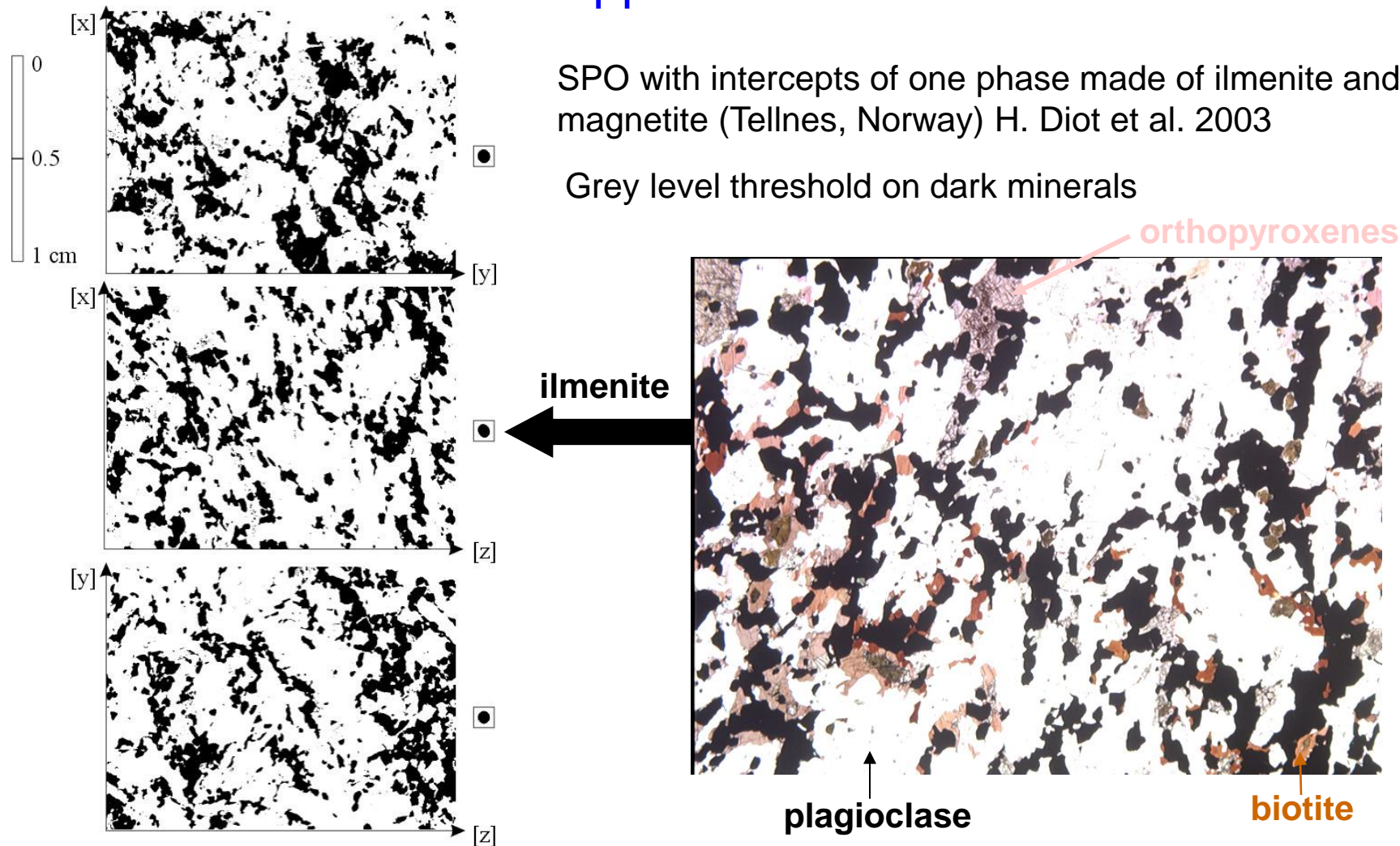


P. Launeau (2004)
 "Mise en évidence des écoulements magmatiques par analyse d'images 2-D des distributions 3-D d'orientations Préférentielles de Formes". *Bull. Soc. Géol. Fr.*, 175, 331-350

Application to ilmenite-rich norite

SPO with intercepts of one phase made of ilmenite and magnetite (Tellnes, Norway) H. Diot et al. 2003

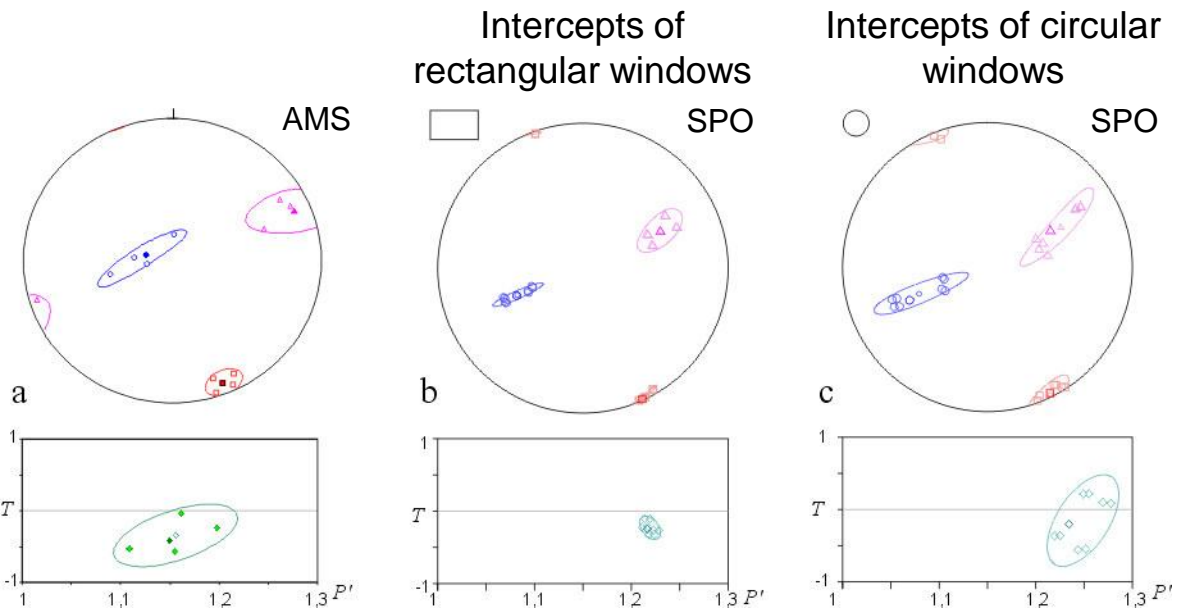
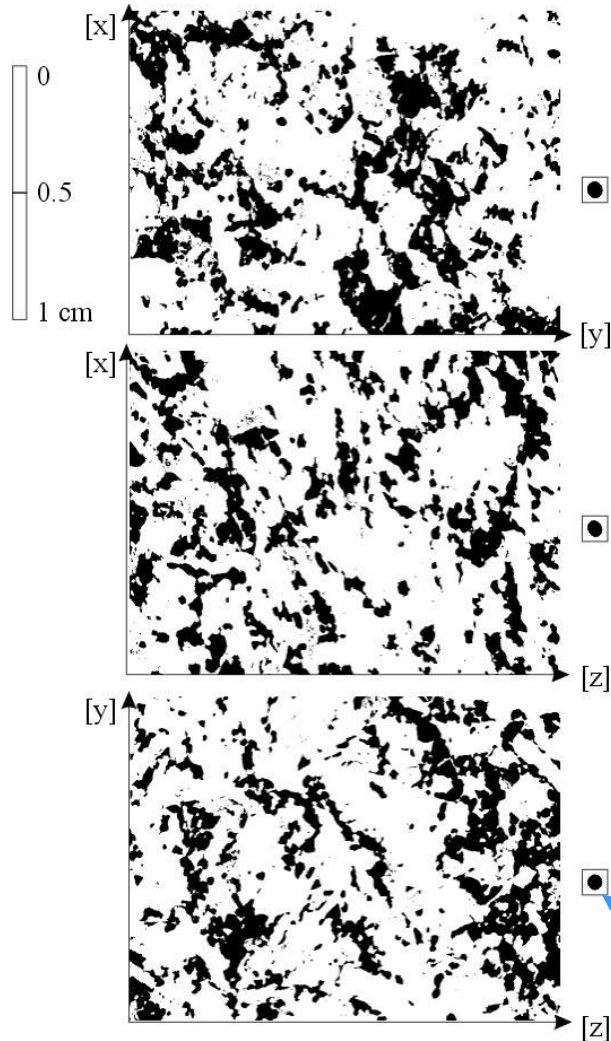
Grey level threshold on dark minerals



Application to ilmenite-rich norite

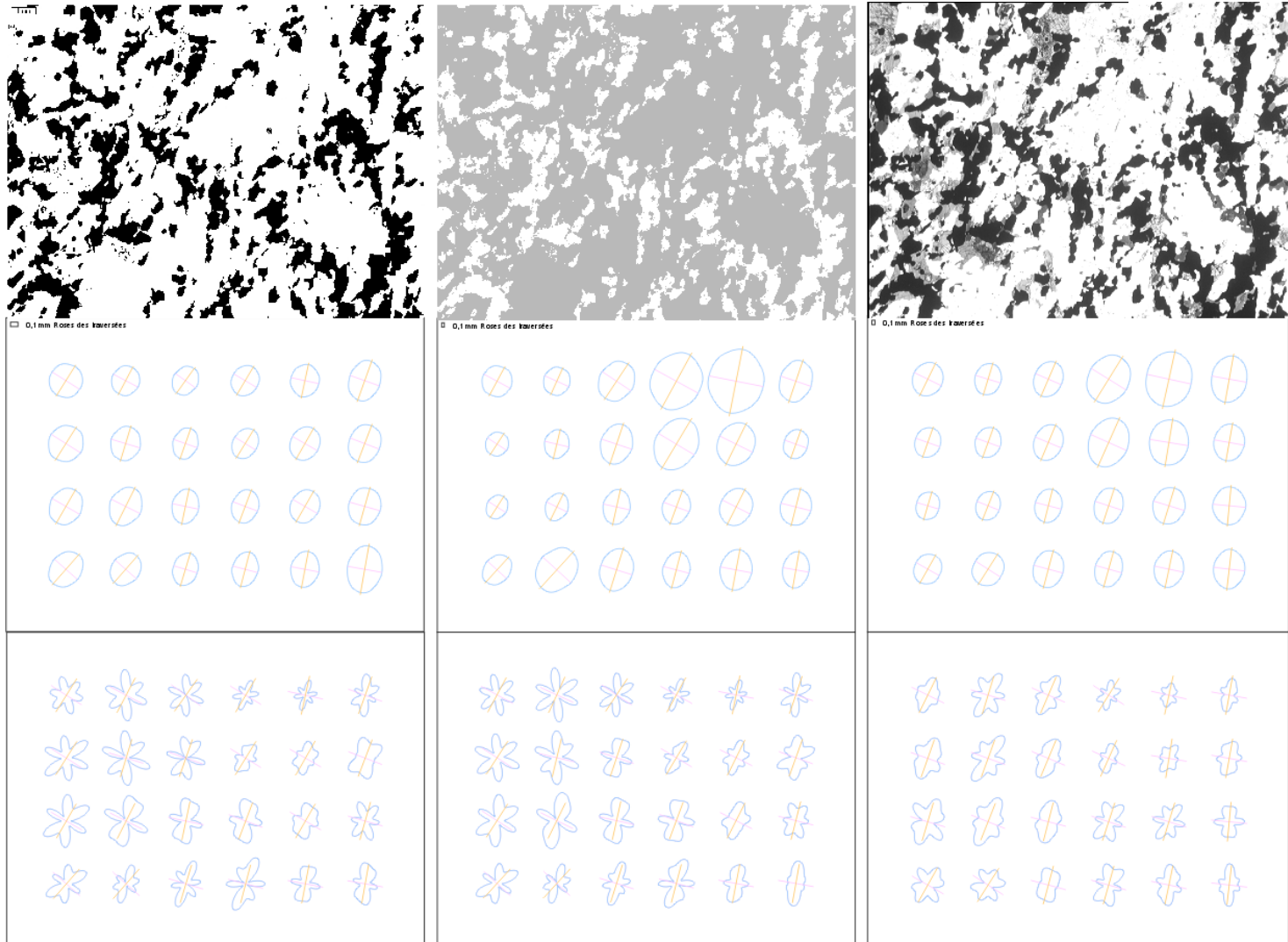
SPO with intercepts of one phase made of ilmenite
(Tellnes, Norway) H. Diot et al. 2003

Grey level threshold on dark minerals

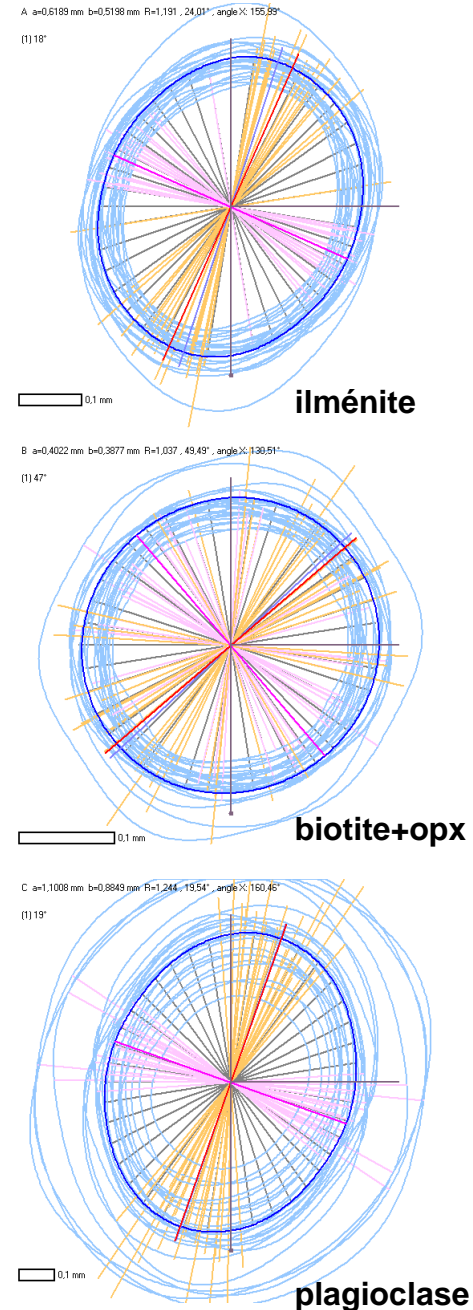
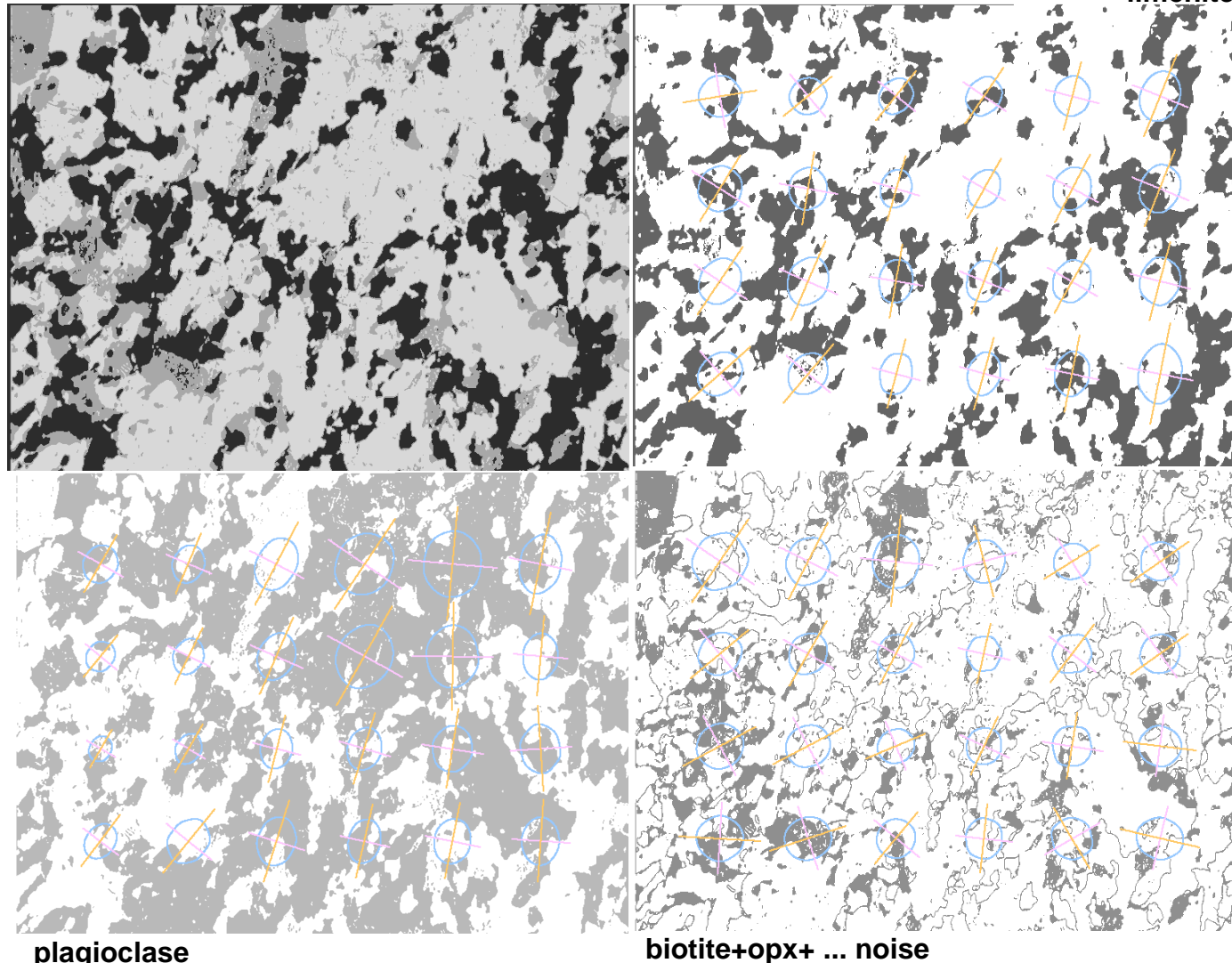


roses of mean intercept length

Application to ilmenite-rich norite



Application to ilmenite-rich norite

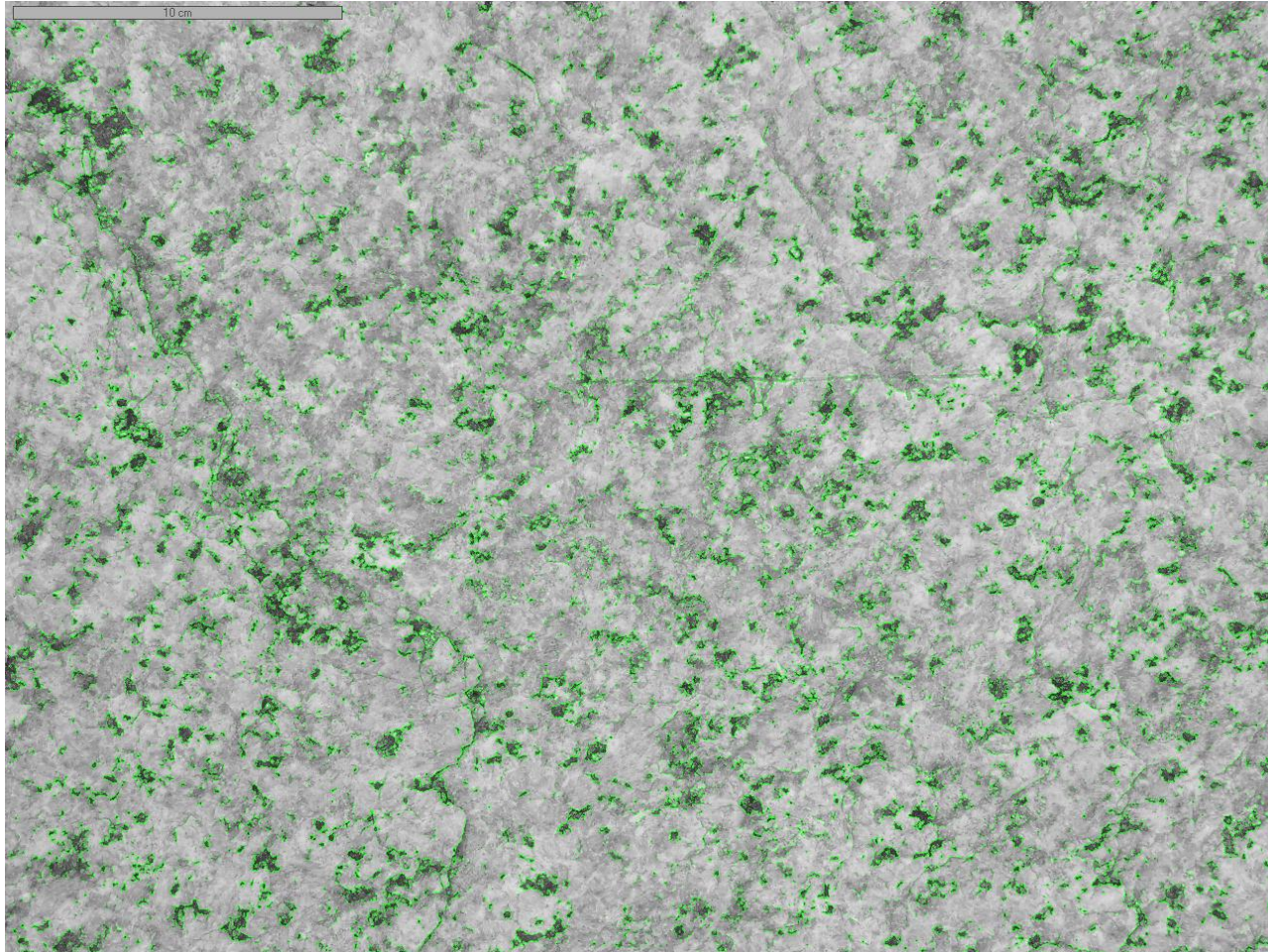


Application to the Pocinhos granite (Brazil)



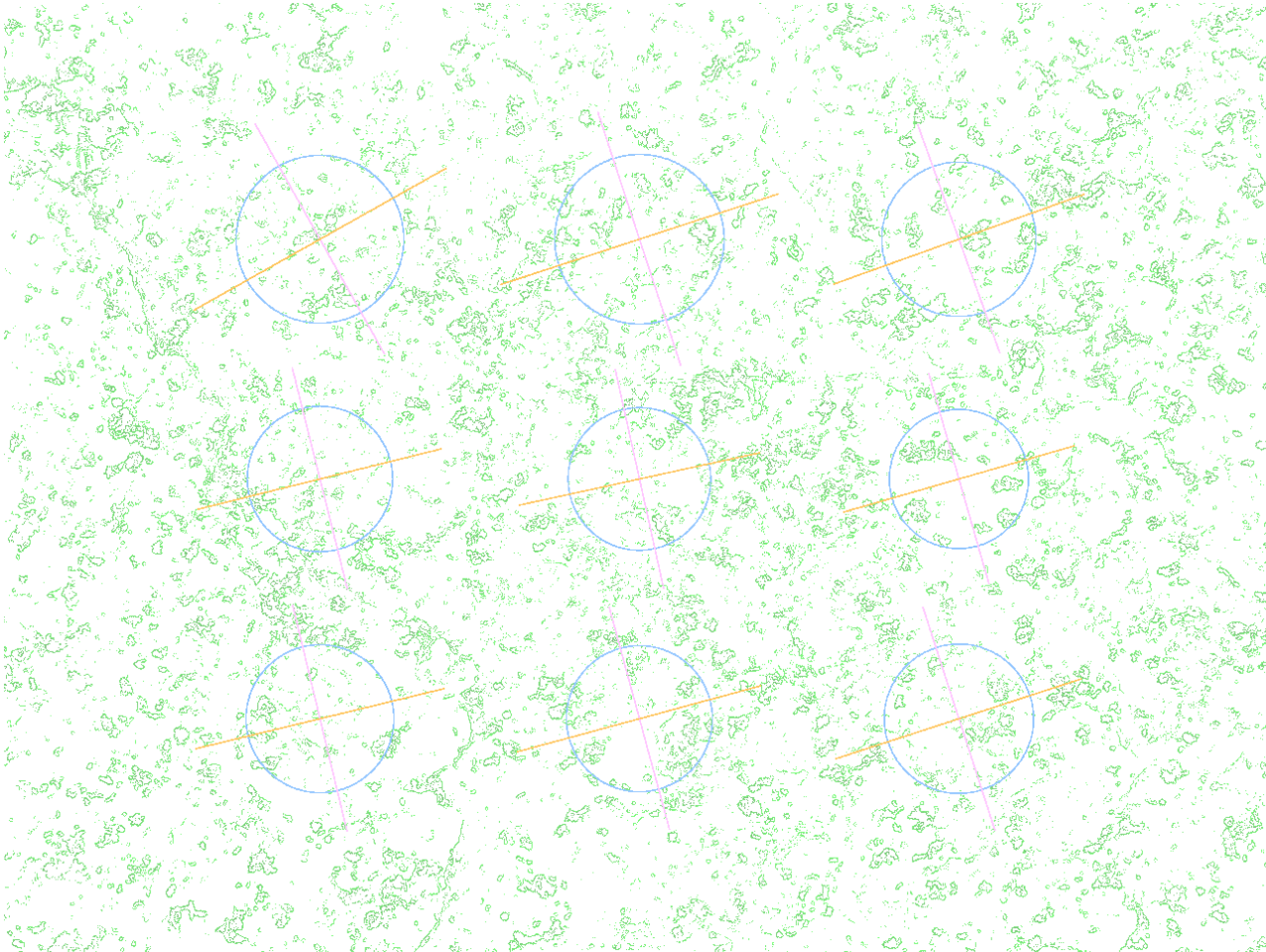
RGB

Application to the Pocinhos granite (Brazil)



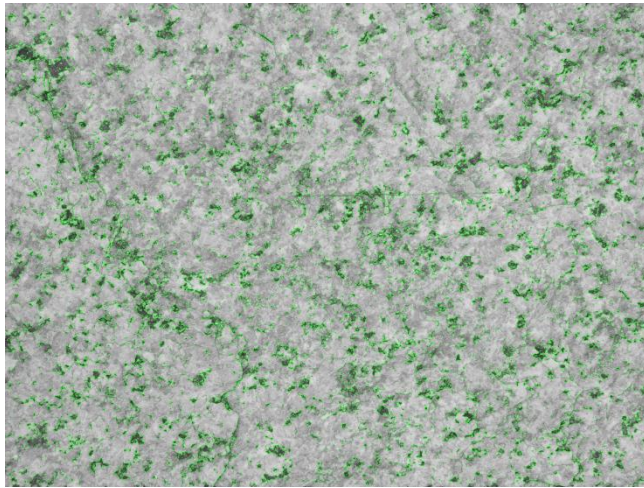
intercepts

Application to the Pocinhos granite (Brazil)



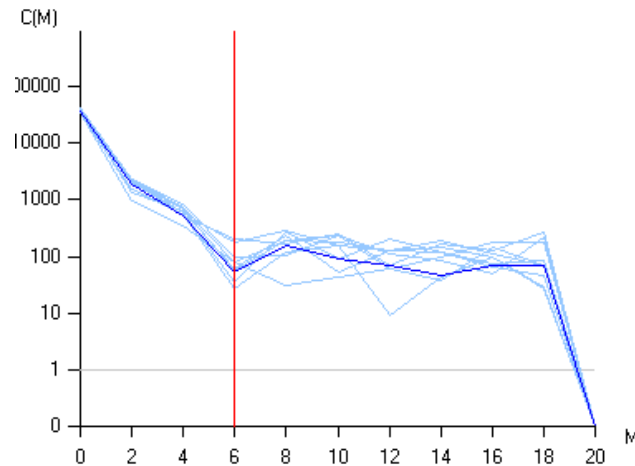
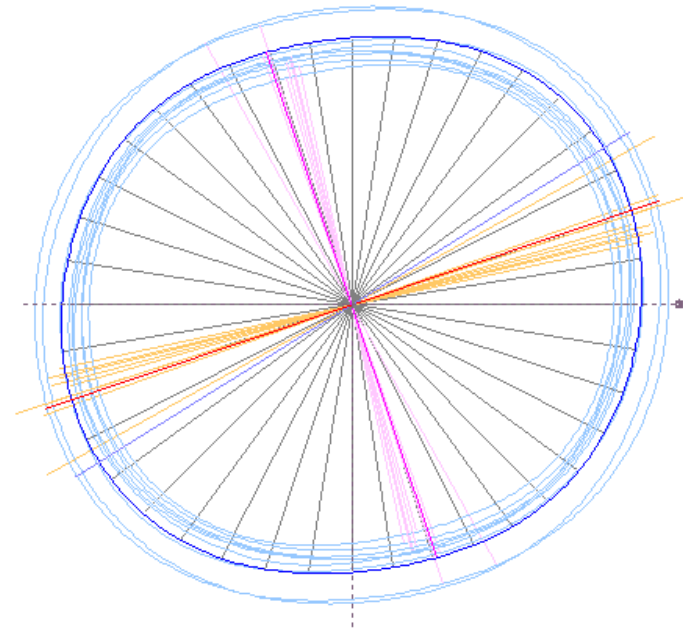
intercepts

Application to the Pocinhos granite (Brazil)

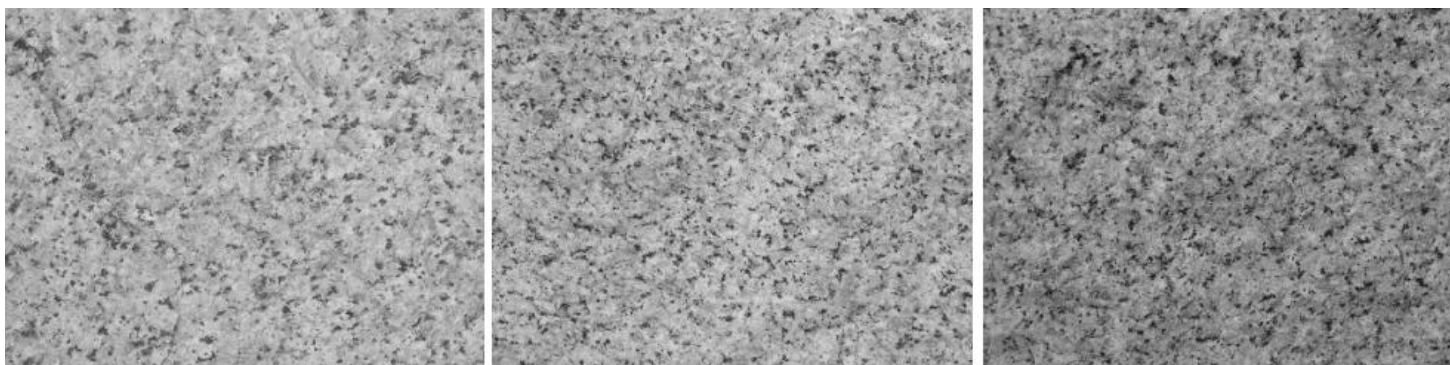


A a=2,7108 cm b=2,4486 cm R=1,107 , 71,20° , angle X: 161,20°

(1) 58°



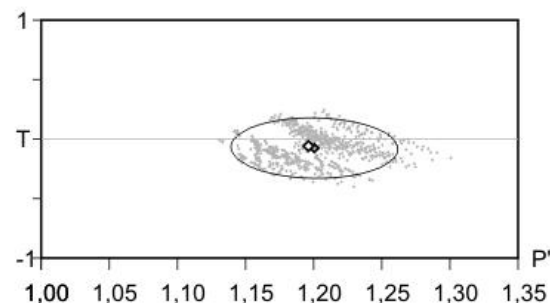
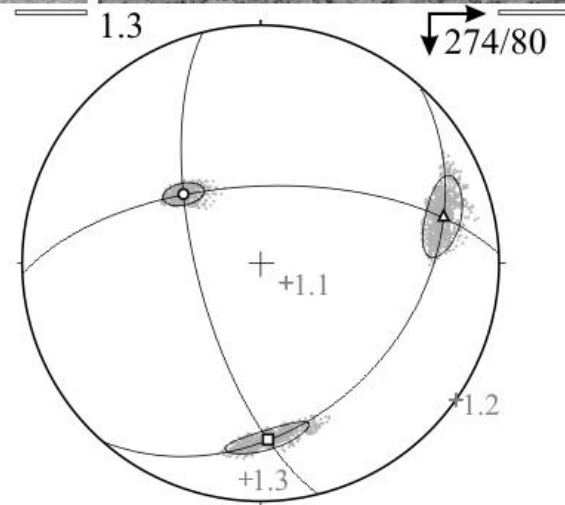
□ 0,1 cm



Application
to the
Pocinhos
granite
(Brazil)

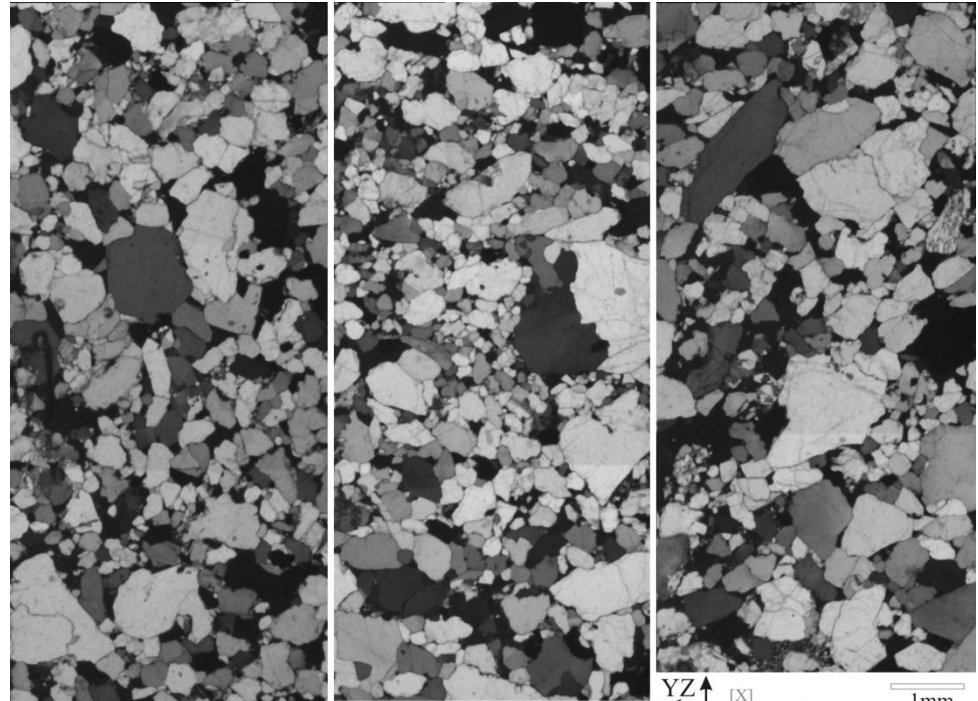
	A	B	C	\sqrt{F}
L. norm	1.095	0.997	0.916	1.4 %
trend	177.6	75.9	311.9	
plunge	26.8	21.9	54.2	
A/C	1.196		Flinn	1.130
A/B	1.099		P'	1.196
B/C	1.088		T	-0.059

	A	B	C	729 combinations	
L. norm	1.098	0.995	0.915	\bar{X}	
	0.030	0.015	0.023	2σ	
trend	177.5	75.9	311.9	\bar{X}	
plunge	26.6	22.0	54.2	\bar{X}	
	13.1	13.4	7.1	$2\sigma_1$	
	3.8	6.5	3.7	$2\sigma_2$	
	\bar{X}	2σ	\bar{X}	2σ	
A/C	1.200	0.061	\sqrt{F}	1.7 %	2.8 %
A/B	1.103	0.040	P'	1.201	0.061
B/C	1.088	0.035	T	-0.076	0.254



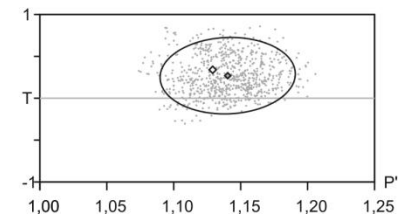
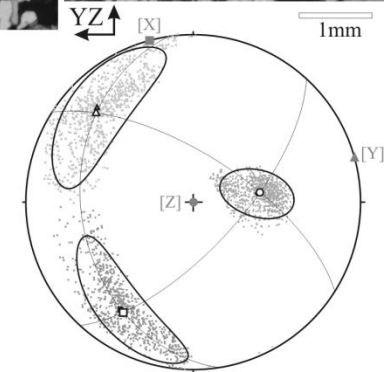
Application to sandstones

Application to maximum polarized light, using Fueten and Goodchild (2001) methodology; thin sections of sandstones from the Devonian beds of the Furnas Formation (Brazil)



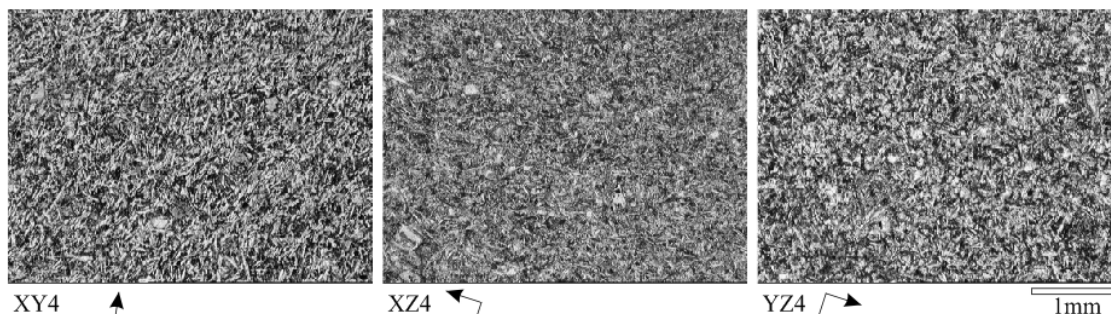
	A	B	C	\sqrt{F}
L. norm	1.054	1.014	0.936	2.3%
trend	212.3	312.4	81.7	
plunge	23.0	22.4	56.9	
A/C	1.127		Flinn	0.482
A/B	1.040		P'	1.129
B/C	1.083		T	0.339

	A	B	C	729 combinations
L. norm	1.060	1.011	0.933	\bar{X}
	0.024	0.019	0.023	2σ
trend	214.1	314.2	82.4	\bar{X}
plunge	23.5	21.0	57.9	\bar{X}
	35.5	35.8	19.1	$2\sigma_1$
	12.8	17.0	11.1	$2\sigma_2$
	\bar{X}	2σ	\bar{X}	2σ
A/C	1.137	0.050	\sqrt{F}	1.9% 2.8%
A/B	1.048	0.035	P'	1.140 0.051
B/C	1.085	0.042	T	0.269 0.457

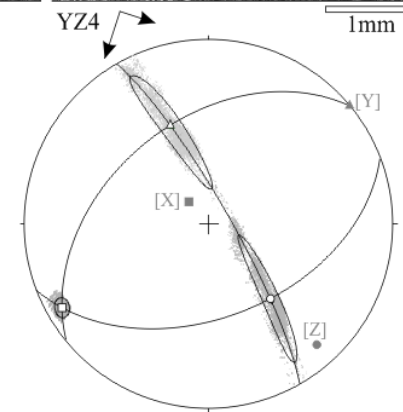


Application to a chilled margin of a diabase dike

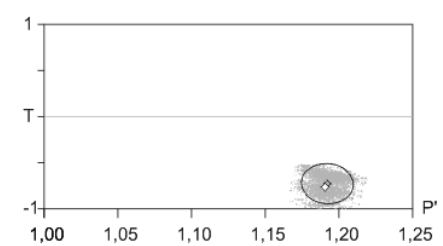
Application to maximum polarized light, using Fueten and Goodchild (2001) methodology; The specimen comes from about 3 cm from the margin of a Mesozoic diabase dike (Rio Cear a-Mirim swarm, NE Brazil)



	A	B	C	\sqrt{F}
L. norm	1.105	0.960	0.942	2.1%
trend	240.2	338.8	140.3	
plunge	9.4	42.3	46.2	
A/C	1.173		Flinn	7.974
A/B	1.151		P'	1.191
B/C	1.019		T	-0.765



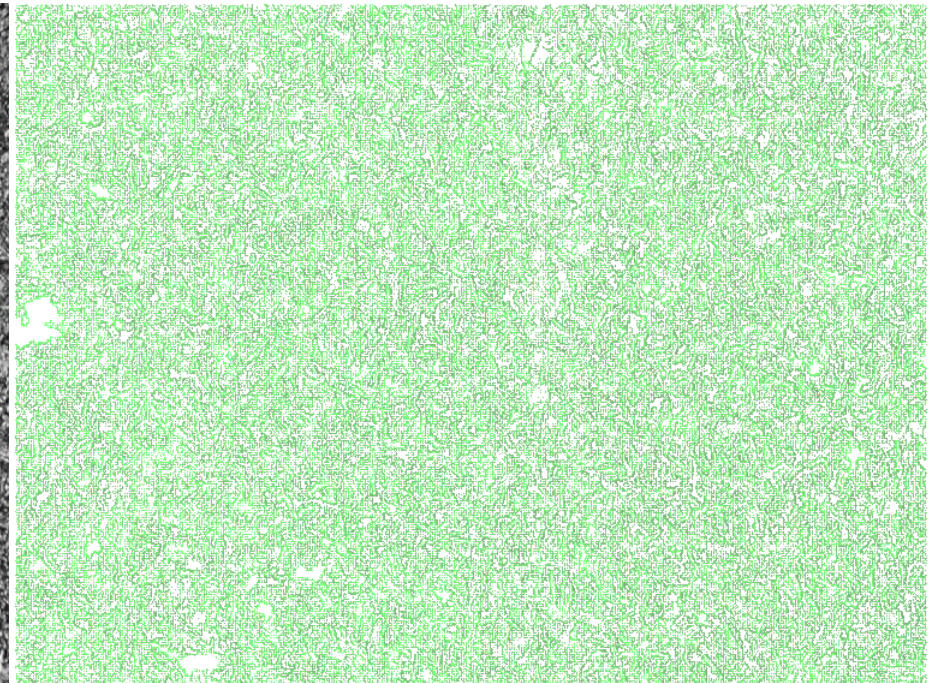
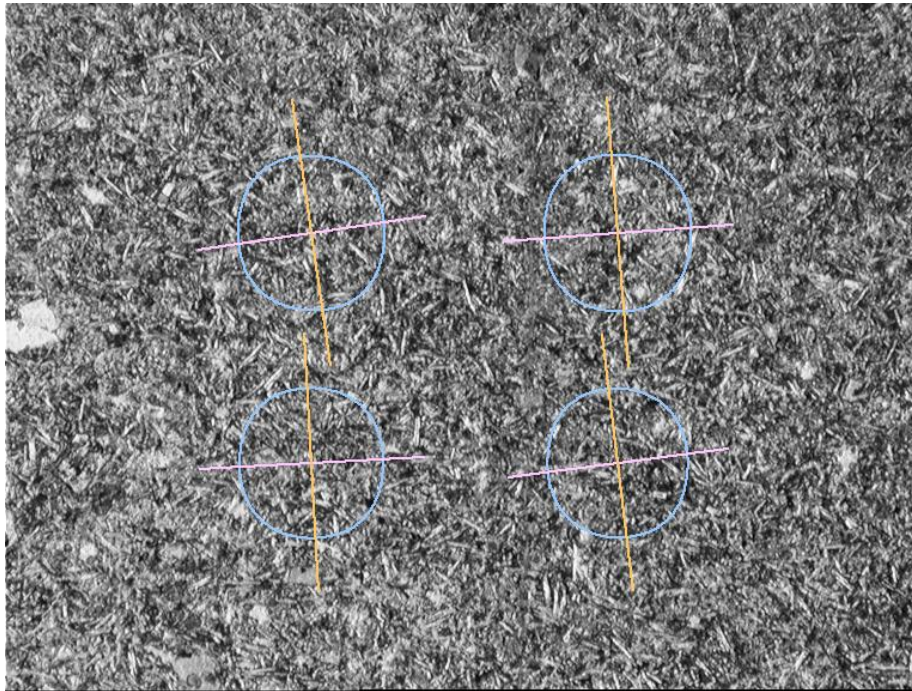
	A	B	C	4096 combinations	
L. norm	1.106	0.962	0.941	\bar{X}	
	0.010	0.010	0.009	2σ	
trend	240.2	339.1	140.6	\bar{X}	
plunge	9.3	43.3	45.1	\bar{X}	
	4.5	33.7	33.7	$2\sigma_1$	
	3.2	4.3	3.5	$2\sigma_2$	
	\bar{X}	2σ	\bar{X}	2σ	
A/C	1.176	0.018	\sqrt{F}	4.6%	1.8%
A/B	1.150	0.019	P'	1.192	0.018
B/C	1.023	0.019	T	-0.727	0.217



Launeau P., Archanjo C. J., Picard D., Arbaret L., Robin P.Y. (2010). Two- and three-dimensional shape fabric analysis by the intercept method in grey levels. Tectonophysics, Volume 492, Issues 1-4, 20 September 2010, Pages 230-239

Application to a chilled margin of a diabase dike

Application to maximum polarized light, using Fueten and Goodchild (2001) methodology; The specimen comes from about 3 cm from the margin of a Mesozoic diabase dike (Rio Cear a-Mirim swarm, NE Brazil)



Cm28-3-XZ.bmp