Design of advanced resolver-to-digital converters

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Introduction

What can be done when the sensor yields two signals proportional to the sine and cosine of the shaft position ?

This happens

- when a resolver is used
- when a magnetic position encoder is used
- when injecting high frequency voltages into salient PMSM machines for sensorless control

This paper both recalls the classical solution, with its **possible settings and performances**, and **two new estimators**.

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The classical angle tracking observer

The classical solution is based on a second-order state space model with a “nearly constant speed” (the acceleration is considered as a random noise) and a linear measurement of the position.

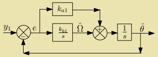
- State space model :

$$\begin{aligned} \dot{X}(t) &= A_1\, X(t) + G_1\, \alpha(t), \text{ with } X = \begin{pmatrix} \theta \\ \Omega \end{pmatrix}, \\ y_1(t) &= C_1\, X(t) + w(t), \text{ with } \Omega = \dot{\theta} \\ A_1 &= \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, C_1^T = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \text{ and } G_1 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \end{aligned}$$

- Luenberger observer

$$\begin{aligned} \dot{\hat{X}}(t) &= A_1\, \hat{X}(t) + K_{c1}\, e(t), \quad \text{with} \quad e(t) = y_1(t) - C_1\, \hat{X}(t) = y_1(t) - \hat{\theta}(t) \\ \text{with } K_{c1}^T &= (k_{a1} \;\; k_{b1}) \text{ and } \hat{X}^T = \begin{pmatrix} \hat{\theta} & \hat{\Omega} \end{pmatrix} \end{aligned}$$

The classical angle tracking observer



The resulting position and speed estimator is equivalent to :

- **an integrator in closed-loop with a PI controller**

$$K_p = k_{a1} \quad \text{and} \quad T_i = k_{a1} / k_{b1}$$

  a common structure in automatic control

- **a filtering process applied to the measured position**

$$\hat{\Theta}(s) = \frac{k_{a1}\, s + k_{b1}}{s^2 + k_{a1}\, s + k_{b1}}\, Y_1(s)$$

$$\hat{\Omega}(s) = \frac{k_{b1}}{s^2 + k_{a1}\, s + k_{b1}}\, s\, Y_1(s)$$

  $\hat{\Theta}(s)$ results from a 2nd-order lowpass filter with a slope of only -20 dB/dec

The classical angle tracking observer

Properties

The classical ATO is unbiased when the speed is constant **but biased when the speed is not constant.**

- if $Y_1(s) = \Theta(s) = \frac{\omega}{s^2}$,

$$\lim_{t \to +\infty} y_1(t) - \hat{\theta}(t) = 0$$

- if $Y_1(s) = \Theta(s) = \frac{\alpha}{s^3}$,

$$\lim_{t \to +\infty} y_1(t) - \hat{\theta}(t) = \frac{\alpha}{k_{b1}}$$

The classical angle tracking observer

Possible settings of the ATO

The setting of the ATO parameters should be deduced **from desired performances in the time domain** rather that in the frequency domain.

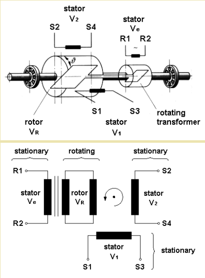
For a linearly increasing speed with acceleration equal to α,

- $k_{b1} = \frac{\alpha}{\theta_{\text{true}} - \hat{\theta}}$ and $k_{a1} = \frac{\Omega_{\text{true}} - \hat{\Omega}}{\theta_{\text{true}} - \hat{\theta}}$
- $k_{b1} = \frac{\alpha}{\theta_{\text{true}} - \hat{\theta}}$ and $k_{a1} = 2m\sqrt{k_{b1}}$,
  where m is a damping ratio. $m = 1.945$ provides a 5 % overshoot when the actual position abruptly changes from 0 to 180°. **A "Butterworth" setting ($m = \sqrt{2}/2$) leads to an overshoot of 20 %** !

The higher the acuracy, the longer the transients

The classical angle tracking observer

resolver :



source :

http ://data.bolton.ac.uk.

nonlinear measurement equation

if the sensor provides two noisy signals

$$\begin{aligned} y_c(t) &= \cos(\theta(t)) + w_c(t) \\ y_s(t) &= \sin(\theta(t)) + w_s(t), \end{aligned}$$

the error term $e(t) = y_1(t) - \hat{\theta}(t)$ should be **simply replaced** by

$$\begin{aligned} \epsilon(t) &= y_s(t)\,\cos(\hat{\theta}(t)) - y_c(t)\,\sin(\hat{\theta}(t)) \\ &= \sin(\theta(t) - \hat{\theta}(t)) + w_s(t)\,\cos(\hat{\theta}(t)) - w_c(t)\,\sin(\hat{\theta}(t)) \\ &\approx \theta(t) - \hat{\theta}(t) + w_s(t)\,\cos(\hat{\theta}(t)) - w_c(t)\,\sin(\hat{\theta}(t)) \end{aligned}$$

The resulting observer **remains stable** [Harnefors2000].

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A new continuous-time third-order ATO

Another angle tracking observer can be derived from on a third-order state space model with a “nearly constant acceleration” (the jerk is considered as a random noise) and a linear position measurement.

- State space model :

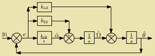
$$
\begin{aligned}
\dot{X}(t) &= A_2\, X(t) + G_2\, \beta(t), \text{ with } X=\begin{pmatrix}\theta\\ \Omega\\ \alpha\end{pmatrix}\\
y_1(t) &= C_2\, X(t) + w(t),\\
A_2 &= \begin{pmatrix}0 & 1 & 0\\ 0 & 0 & 1\\ 0 & 0 & 0\end{pmatrix}, C_2^T=\begin{pmatrix}1\\0\\0\end{pmatrix} \text{ and } G_2=\begin{pmatrix}0\\0\\1\end{pmatrix},
\end{aligned}
$$

- Luenberger observer

$$
\dot{\hat{X}}(t) = A_2\, \hat{X}(t) + K_{c2}\, e(t), \quad \text{with} \quad e(t) = y_1(t) - C_2\, \hat{X}(t) = y_1(t) - \hat{\theta}(t)
$$

$$
\text{with } K_{c2}^T = \begin{pmatrix}k_{a2} & k_{b2} & k_{c2}\end{pmatrix} \text{ and } \hat{X}^T=\begin{pmatrix}\hat{\theta} & \hat{\Omega} & \hat{\alpha}\end{pmatrix}
$$

A new continuous-time third-order ATO



The resulting position, speed and acceleration estimator is equivalent to :

- **an double integrator in closed-loop with a PID controller**

$$K_p = k_{b2}, \quad T_i = \frac{k_{b2}}{k_{c2}} \quad \text{and} \quad T_d = \frac{k_{a2}}{k_{b2}}$$

  still a common structure in automatic control

- **a third-order filtering process applied to the measured position**

$$\begin{aligned}
\hat{\Theta}(s) &= \frac{k_{a2}\, s^2 + k_{b2}\, s + k_{c2}}{s^3 + k_{a2}\, s^2 + k_{b2}\, s + k_{c2}}\, Y_1(s) \\
\hat{\Omega}(s) &= \frac{(k_{b2}\, s + k_{c2})}{s^3 + k_{a2}\, s^2 + k_{b2}\, s + k_{c2}}\, s\, Y_1(s) \\
\hat{\alpha}(s) &= \frac{k_{c2}}{s^3 + k_{a2}\, s^2 + k_{b2}\, s + k_{c2}}\, s^2\, Y_1(s)
\end{aligned}$$

  $\hat{\Theta}(s)$ results from a third-order lowpass filter with a slope of only -20 dB/dec

A new continuous-time third-order ATO

Properties

This estimator is unbiased when the speed is constant **but also when the speed is linearly increasing**

- if $Y_1(s) = \Theta(s) = \frac{\omega}{s^2}$,
$$\lim_{t \to +\infty} y_1(t) - \hat{\theta}(t) = 0$$
- if $Y_1(s) = \Theta(s) = \frac{\alpha}{s^3}$,
$$\lim_{t \to +\infty} y_1(t) - \hat{\theta}(t) = 0$$

This estimator provides an improved position and speed estimation during transients.

A new continuous-time third-order ATO

Possible settings of this ATO

Placing the poles of the transfer functions at $-K/T$, $-1/T + \jmath\psi/T$ and $-1/T - \jmath\psi/T$ to obtain a desired settling time, a desired peak overshoot and a desired natural frequency of oscillation.

$$k_{a2} = \frac{K+2}{T}, \quad k_{b2} = \frac{\psi^2 + 2K + 1}{T^2}, \quad k_{c2} = \frac{K\left(\psi^2+1\right)}{T^3}$$

For example, simulation results show that $\psi = 3\pi/2$ and $K = 39.04$ leads to a peak overshoot of 10 % when the actual position abruptly changes from 0 to 180°.

A “Butterworth” setting ($k_{a2} = 2/T_c$, $k_{b2} = 2/T_c^2$, $k_{c2} = 1/T_c^3$) would lead to an overshoot of 30.9 **%** !

The higher the acuracy, the longer the transients

A new discrete-time third-order ATO

Another angle tracking observer can be derived from on a discrete-time third-order state space model, using a **statistical state-space estimator**.

State space model : linear transition equation and nonlinear measurement equation

$$
\begin{aligned}
X_{[k+1]} &= A_3\, X_{[k]} + G\, v_{[k]} \\
Y_{[k+1]} &= \begin{pmatrix} y_{c\,[k+1]} \\ y_{s\,[k+1]} \end{pmatrix} = \begin{pmatrix} y_c\big((k+1)\,T_s\big) \\ y_s\big((k+1)\,T_s\big) \end{pmatrix} = \mathcal{H}(X_{[k+1]}) + W_{[k+1]}, \\
\text{with } X_{[k]} &= \begin{pmatrix} x_{1\,[k]} \\ x_{2\,[k]} \\ x_{3\,[k]} \end{pmatrix} = \begin{pmatrix} \theta(k\,T_s) \\ T_s\,\Omega(k\,T_s) \\ T_s^2\,\alpha(k\,T_s) \end{pmatrix},\ A_3 = \begin{pmatrix} 1 & 1 & 1/2 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}, \\
G^T &= \begin{pmatrix} 1/6 & 1/2 & 1 \end{pmatrix},\ v[k] = T_s^3\,\beta(k\,T_s), \\
\mathcal{H}(X_{[k]}) &= \begin{pmatrix} \cos(x_{1\,[k]}) \\ \sin(x_{1\,[k]}) \end{pmatrix} \text{ and } W_{[k]} = \begin{pmatrix} w_{c\,[k]} \\ w_{s\,[k]} \end{pmatrix}
\end{aligned}
$$

A new discrete-time third-order ATO

Another angle tracking observer can be derived from on a discrete-time third-order state space model, using a **statistical state-space estimator**.

nonlinear Kalman estimator : using a third order Taylor expansion of the measurement function $\mathcal{H}$, a simple and computationally efficient **time-invariant** state estimator can be designed :

$$\begin{aligned} X_{p[k]} &= A_3 \, X_{e[k-1]} \\ X_{e[k]} &= X_{p[k]} + K_{\text{lin}} \left(y_{s[k]} \cos(\hat{\theta}_{p[k]}) - y_{c[k]} \sin(\hat{\theta}_{p[k]}) \right) \end{aligned}$$

where K_{lin} is derived from the variances of the state noise q and the measurement noise r. This is a **new and very surprising result**, because “extended” Kalman filters are seldom simple.

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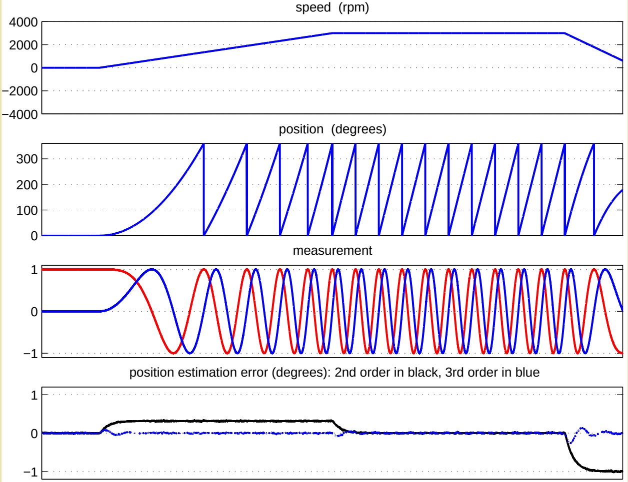
Simulation Results



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Conclusion

- Performances, setting, and digital implementation of the classical ATO
- Performances, setting, and digital implementation of a new continuous-time third-order ATO
- New discrete-time estimator based on estimation theory
- MATLAB/SIMULINK files available at http ://www.univ-nantes.fr/auger-f

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Conclusion

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- Thanks for your attention
- Any questions ?

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